

MULTI-DISCIPLINARY SYSTEM RELIABILITY ANALYSIS

FINAL REPORT

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Abstract

The objective of this study is to develop a new methodology for estimating the reliability of engineering systems that encompass multiple disciplines. The methodology is formulated in the context of the NESSUS probabilistic structural analysis code, developed under the leadership of NASA Lewis Research Center. The NESSUS code has been successfully applied to the reliability estimation of a variety of structural engineering systems. This study examines whether the features of NESSUS could be used to investigate the reliability of systems in other disciplines such as heat transfer, fluid mechanics, electrical circuits etc., without considerable programming effort specific to each discipline. In this study, the mechanical equivalence between system behavior models in different disciplines are investigated to achieve this objective. A new methodology is presented for the analysis of heat transfer, fluid flow, and electrical circuit problems using the structural analysis routines within NESSUS, by utilizing the equivalence between the computational quantities in different disciplines. This technique is integrated with the fast probability integration and system reliability techniques within the NESSUS code, to successfully compute the system reliability of multi-disciplinary systems. Traditional as well as progressive failure analysis methods for system reliability estimation are demonstrated, through a numerical example of a heat exchanger system involving failure modes in structural, heat transfer and fluid flow disciplines.

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CHAPTER I

INTRODUCTION

The purpose of this study is to develop a methodology to estimate the reliability of engineering systems that encompass several disciplines. The methodology is implemented using the NESSUS probabilistic analysis code, which has mostly been applied exclusively in the discipline of structural engineering. In order to apply the NESSUS probabilistic structural analysis code to analyze a multi-disciplinary engineering system, the equivalences between system behavior models in different disciplines are investigated, and the effect of physical interaction among the failure modes is quantified in this study.

System reliability analysis is a method of estimating the effects of uncertainties in an engineering system on the probability of successful performance. Usually, an engineering system consists of multiple subsystems and components, which may require the knowledge of different disciplines of engineering. Such disciplines may include structural engineering, mechanical engineering, heat transfer theory, fluid mechanics, electrical engineering, etc. Such a system is called a multi-disciplinary engineering system. The reliability analysis of any engineering system usually begins with the identification and reliability computation of individual failure modes within the system. Then the reliability analysis of the overall system can be carried out.

Traditionally, reliability methods have primarily concentrated on failures in one

particular discipline, e.g. structural analysis, not on an overall system which consists of multiple disciplines. Furthermore, conventional methods of system reliability estimation usually only consider the statistical correlation between individual failure events, ignoring the fact that more often than not, those individual failure modes also have a physical correlation. This leads to inaccuracy in system reliability estimation.

The method presented in this report uniquely computes the failure probability interactions between different modes and overall system failure probability through the imposition of one failure mode on another field and reanalysis of the latter. This method is used to compute the probabilities of critical system failure events after accounting for the contributing non-critical failure modes in all different fields. However, it is not an easy task to estimate the reliability interactions between different failure modes. The success of such a method primarily depends on the availability of effective reliability tools. The software system NESSUS developed under the leadership of NASA Lewis Research Center is uniquely suited for this purpose. Currently, this code has been applied primarily to the structural engineering problems. In order to perform system reliability analysis including the interactive failure modes, this study uniquely develops behavior analogies between the structural model and heat transfer model, and between the structural model and fluid mechanics model. By doing so, the probability estimation of heat transfer and fluid mechanics failures can be pursued similarly to structural reliability analysis.

The objective of this research project is to develop a method, using system reliability theory, for the reliability estimation of multi-disciplinary engineering systems. The method is implemented on the software system NESSUS (Numerical Evalu-

ation of Stochastic Structures under Stress) developed by NASA Lewis Research Center. An example application to a three-discipline system involving mechanical stress-strain behavior, heat transfer and fluid mechanics is provided. In order to compute the individual failure mode probability of non-structural problems such as heat transfer and fluid mechanics within NESSUS, it is necessary to develop a new methodology for the analysis of heat transfer problems using the concept of equivalence between the computational quantities in structural analysis, such as stiffness, displacement vector, load vector, etc. and similar quantities such as conduction, temperature distribution and heat flux in heat transfer theory, flow velocity, pressure and flow factor in fluid mechanics. This is the first important contribution of this study.

The second important contribution is the method for the computation of the physical dependence of critical failure mode probabilities on non-critical failure modes in various disciplines. This involves the imposition of the non-critical modes and reanalysis of the system with appropriate discipline equivalences, for various levels of progressive damage. The combination of these two ideas - inter-disciplinary analogies and physical failure mode correlation - makes a reliability analysis program such as NESSUS very powerful for application to a variety of multi-disciplinary systems.

The concepts and methods discussed above are examined in detail in the next four chapters of this report. In Chapter II, the basic reliability analysis concepts for individual component-level and system-level events are reviewed, and their implementation in the NESSUS program is described. In Chapter III, the behavior analogies between the structural analysis model and heat transfer problem, and be-

tween the structural analysis model and fluid mechanics model are developed. The finite element numerical examples with NESSUS/FEM are demonstrated for this concept. Chapter IV consists of two major parts: in the first part, the failure probability analyses for individual events including structural, heat transfer and fluid flow failure modes are performed using NESSUS; in the second part, the system failure probability is studied. The effect of non-critical failure events of heat transfer and fluid mechanics upon a critical structural failure event is investigated, followed by system reliability analysis with the consideration of physically correlated component-level events. A numerical example of system reliability analysis of a multi-disciplinary system consisting of structural, heat transfer and fluid mechanical modes is demonstrated. The conclusions and recommendations of the study are summarized in Chapter V.

CHAPTER II

SYSTEM RELIABILITY ANALYSIS

Individual failure modes and effects

An engineering system consists of a number of functional components. Before the system-level analysis begins, the modes of failure for individual components should be specified. The analyses of the failure modes and effects can be carried out by starting at the component level and expanding upward to the whole system. A failure mode is the manner by which a failure is observed. All units in a system are designed to fulfill one or more functions. A failure is thus defined as non-fulfillment of one of these functions. Analytically, each failure mode has a corresponding limit state which separates the design space into “failure” and “safe” regions. The probability of failure, P_f , is denoted as

$$P_f = P[g \leq 0] \quad (1)$$

where g is the value of the performance function $g(X)$. The limit-state is denoted by the equation $g(X) = 0$.

An exact solution of P_f can be obtained by the integration of the multiple integral denoted as

$$P_f = \int_{g(X) \leq 0} f_X(x) dx \quad (2)$$

where $f(X)$ is the joint probability density function of the vector of uncertain variables \tilde{X} .

In general, the solution of this multiple integral is too complicated to obtain. This is not only because the individual distributions are not always available but also because the integral is multi-dimensional for a realistic problem and is difficult to evaluate. Therefore, for practical purposes, efficient approximate analysis tools are needed.

Fig. 1 illustrates the concept of the first-order approximation to the limit state for an estimate of the failure probability.

The uncertain variables (\tilde{X}) are all transformed to equivalent uncorrelated standard normal variables (u). The most probable point *MPP* of the limit state is defined at the minimum distance β from the origin to the limit state surface. Therefore, the first-order estimate of the failure probability is

$$P_f = \Phi(-\beta) \quad (3)$$

where Φ is the distribution function of a standard normal variables.

In the NESSUS computer code, this is referred to as the Fast Probability Integration (*FPI*) method. The limit state is constructed as:

$$g = Z(X) - Z_0 = 0 \quad (4)$$

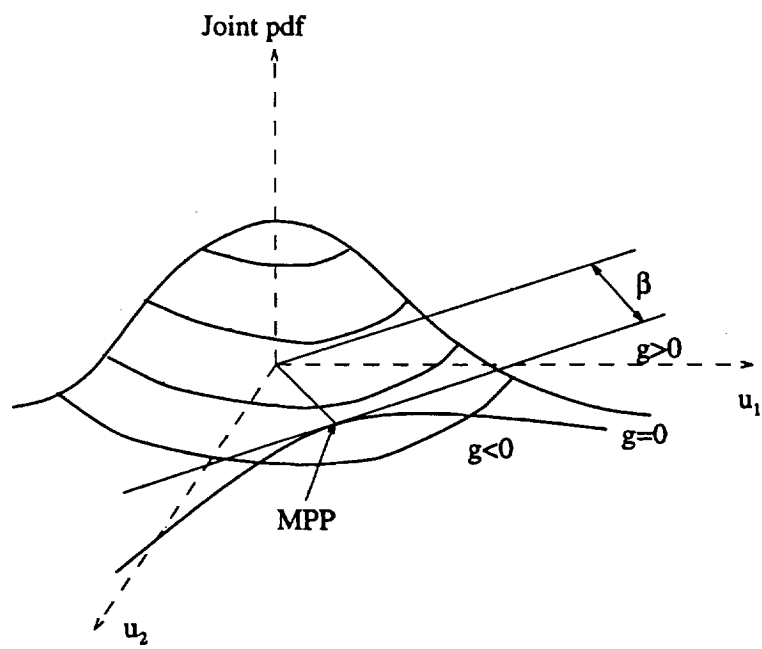


Figure 1: Failure Probability Estimation.

where Z_0 is a real value of the random variable $Z(X)$, which is a performance function or a response function, such as stress, displacement, temperature, etc.

$$Z(X) = Z(X_1, X_2, \dots, X_n) \quad (5)$$

where $X_i (i = 1, 2, \dots, n)$ are the input random variables.

The NESSUS program searches for the *MPP* by computing the sensitivities of the limit state to the random variables using iterative perturbation (in NESSUS/PFEM), and using these sensitivities to obtain a mean value first order (*MVFO*) or second-order (*MVSO*) estimate of the failure probability (in NESSUS/FPI).

By using a first or second-order Taylor's series expansion around the *MPP*, u^* , the exact $g(u)$ -function is replaced by the first-order polynomial, $g_1(u)$,

$$g_1(u) = a_0 + \sum_{i=1}^n a_i(u_i - u_i^*) \quad (6)$$

or a second-order polynomial, $g_2(u)$,

$$g_2(u) = a_0 + \sum_{i=1}^n a_i(u_i - u_i^*) + \sum_{i=1}^n b_i(u_i - u_i^*)^2 + \sum_{i=1}^n \sum_{j=1}^{i-1} C_{ij}(u_i - u_i^*)(u_j - u_j^*) \quad (7)$$

where the coefficients can be obtained by perturbation.

Once these functions are obtained, the *MPP* is found. The probability of failure can be computed easily using Equation 3. This is the mean value first order (*MVFO*) estimate of the failure probability. This is improved using the Advanced Mean Value (*AMV*) analysis. Point probability estimate is made using specific limits for Z_0 , and the cumulative distribution function (CDF) is obtained by varying Z_0 .

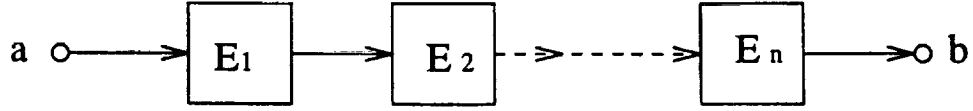


Figure 2: Series System Structure

NESSUS/FEM employs innovative finite element technology and solution strategies. It provides a choice of algorithms for the solution of static and dynamic problems, both linear and nonlinear, together with an interactive perturbation analysis algorithm to evaluate the sensitivity of the response to small variations in one or more user-defined random parameters.

NESSUS/FPI (Fast Probability Integrator) is used to evaluate structural response cumulative distribution functions (CDF). There are two methods in the code, the first-order reliability method and the advanced first-order reliability method. In general, the structural performance or response functions (e.g., stresses, displacements, vibration frequencies) are implicitly defined and each function evaluation may require intensive computation. The AMVFO (Advanced Mean Value First Order) method reduces the computational burden and is the main probabilistic tool in NESSUS. NESSUS/PFEM automates the AMV procedure by integrating the FPI code and the FEM code.

System and component-level failure modes

After the individual reliability analysis is completed, one then proceeds to the system or subsystem level analysis. System failure may occur due to a combination of any of the individual component failure modes. Many physical systems that are composed of multiple components can be classified as series-connected or parallel-connected systems, or combinations of series and parallel conditions. Description of these simple system structures is as follows.

- Series System

A system that is functioning if and only if all of its n components are functioning is called a series system structure. Fig. 2 illustrates such a system.

If E_i denotes the failure mode i , then the failure of a series system is the event

$$E_f = E_1 \cup E_2 \cup \dots \cup E_n \quad (8)$$

Then the failure probability of the system is

$$P_f = P(E_f) \quad (9)$$

If each failure mode E_i is represented by a limit state $g(X) = 0$ in basic variable space, the failure probability can be obtained by the integration denoted as

$$P_f = \int_{\Omega \in X} \dots \int f_X(x) dx \quad (10)$$

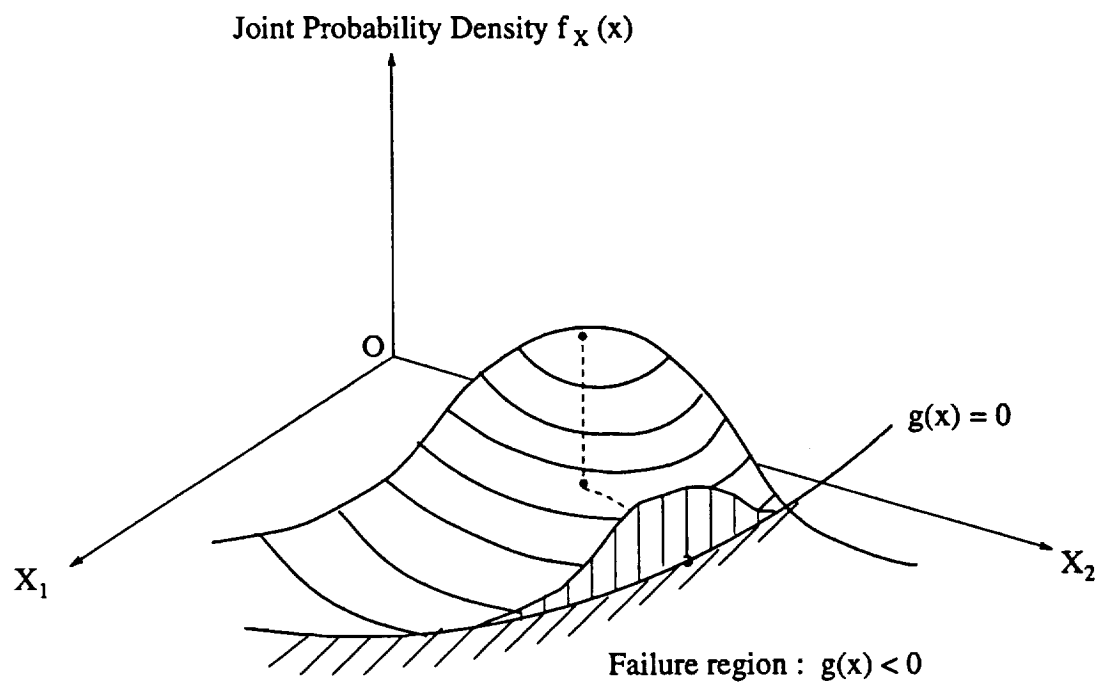


Figure 3: Basic reliability problem in two dimensions

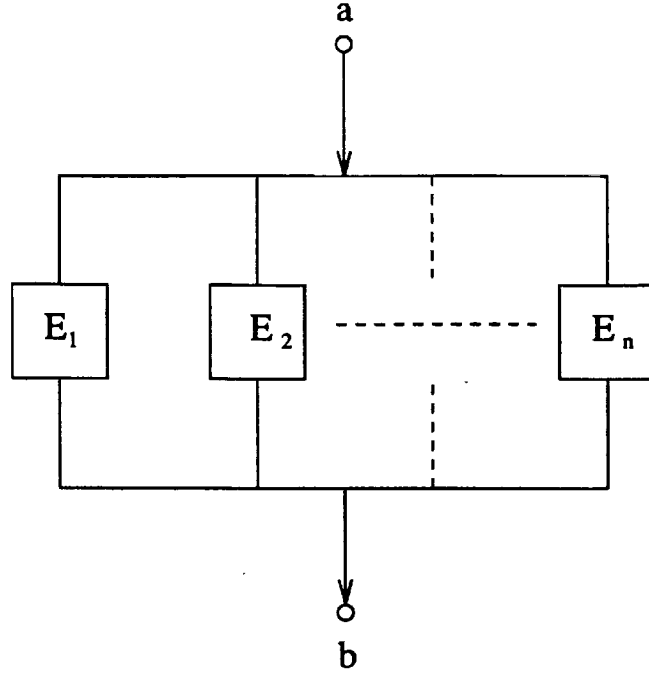


Figure 4: Parallel System Structure

where X represents the vector of all the basic random variable (loads, material properties, etc.) and Ω is the domain in X defining failure of the system.

This is defined in terms of the various failure modes as $g_i(X) \leq 0$. In two-dimensional X space, expression (10) is defined in Fig. 3.

- Parallel System

A system that is functioning if at least one of its n components is functioning is called a parallel system structure. A parallel structure of order n is illustrated in Fig. 4.

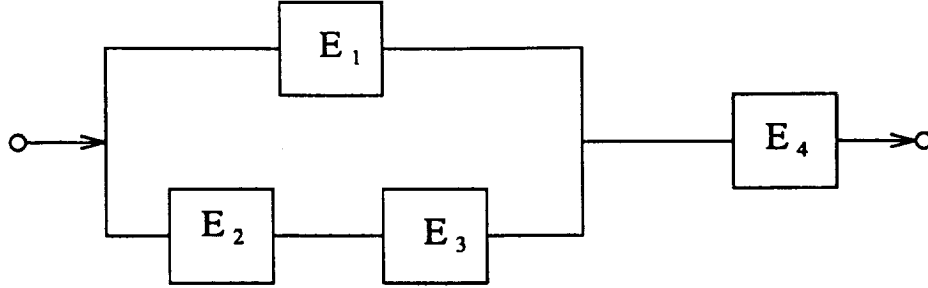


Figure 5: Combined System Structure

In this case, the system failure event can be written

$$E_f = E_1 \cap E_2 \cap \dots \cap E_n \quad (11)$$

- Combined Series-Parallel System Structure

This refers to systems which are a combination of series and parallel structures.

Fig. 5 shows an example of such systems.

The failure event of this system is written, for example, as

$$E_f = [E_1 \cap (E_2 \cup E_3)] \cup E_4 \quad (12)$$

It should be noted that not all engineering systems can be represented simply as described above. Practical systems may be more complex and need more effort to model.

System reliability computation

In NESSUS, two methods have been implemented for system reliability computation [4]: (1) probabilistic fault tree analysis combined with importance sampling (Torng et al, 1992) and (2) a structural reanalysis procedure to accurately estimate the failure regions for various critical failure modes affected by progressive damage (Mahadevan et al, 1992).

Consider an engineering system subject to a sequence of loads (duty cycles) and which may fail in any one (or more) of a number of possible failure modes under any one load in the loading sequence. The total probability of the system failure may then be expressed in terms of the individual mode failure probabilities as

$$P_f = P(E_1) \cup P(E_2 \cap S_1) \cup P(E_3 \cap S_2 \cap S_1) \cup P(E_4 \cap S_3 \cap S_2 \cap S_1) \cup \dots \quad (13)$$

where E_i denotes the “failure of the system due to failure in i th mode and S_i denotes the complementary “survival event of the i th mode.

Since $P(E_2 \cap S_1) = P(E_2) - P(E_2 \cap E_1)$, ..., Eq. 13 may be written also as

$$\begin{aligned} P_f = & P(E_1) + P(E_2) - P(E_1 \cap E_2) + P(E_3) - P(E_1 \cap E_3) \\ & - P(E_2 \cap E_3) + P(E_1 \cap E_2 \cap E_3) + \dots \end{aligned} \quad (14)$$

where $(E_1 \cap E_2)$ is the event that failure occurs in both modes 1 and 2, etc.

Since it is not always an easy task to determine the joint probabilities of more than two failure modes, the following approximation methods can be used to predict the system reliabilities.

- First-order bounds

The probability of failure for the system can be expressed as $P_f = 1 - P(S)$, where $P(S)$ is the probability of survival. For independent failure modes, $P(S)$ can be represented by the product of the mode survival probabilities, or, noting that $P(S_i) = 1 - P(E_i)$, by

$$P_f = 1 - \prod_{i=1}^n [1 - P(E_i)] \quad (15)$$

where, as before, P_f is the probability of failure in mode i . This result can be shown to be identical with Eq. 14. It follows directly from Eq. 14 that, if $P(E_i) \ll 1$, then Eq.15 can be approximated by [Freudenthal et al., 1966]

$$P_f = \sum_{i=1}^n P(E_i) \quad (16)$$

In the case where all failure modes are fully dependent, it follows directly that the weakest failure mode will always govern system failure, irrespective of the random nature of the strength. Hence

$$P_f = \max_{i=1}^n [P(E_i)] \quad (17)$$

Equations 15 or 16 and 17 can be used to define relatively crude bounds on the failure probability of any system of the series types when the failure modes are neither completely independent nor fully dependent. These are Cornell's first-order bounds:

$$\max_{1 \leq i \leq n} P(E_i) \leq P(\cup_{i=1}^n E_i) \leq \sum_{i=1}^n P(E_i) \quad (18)$$

- Second-order bounds

For some practical systems, the above first-order bounds may be too wide to be meaningful. For more accurate estimation, second-order bounds have been developed. There are various second-order bounds in the literature [Kounias, 1968; Vanmarcke, 1973; Hunter, 1976; Ditlevsen, 1979]. Cruse et al (1992) derived second-order bounds which are independent of any ranking of the failure events[1]. The upper bound is

$$P(\cup_{i=1}^n E_i) \leq \left\{ \sum_{i=1}^n P(E_i) - \max \left[\sum_{i=2}^n \max_{j < i} P(E_i E_j), \max_{1 \leq i \leq n} \sum_{i=1, j \neq i}^n P(E_i E_j) \right] \right\} \quad (19)$$

The lower bound is

$$P(\cup_{i=1}^n E_i) \geq \max_{1 \leq j \leq n} \left\{ P(E_j) + \sum_{i=1, i \neq j}^n \max \left[\left(P(E_i) - \sum_{k=1, k \neq i}^{\max(i,j)} P(E_i E_k) \right); 0 \right] \right\} \quad (20)$$

Utilization of the second-order bounds requires evaluation of terms of the form $P(E_i E_j)$ where E_i denotes the event "failure in limit state i ". The intersection terms refer to domains such as Ω_1 shown bounded by the non-linear limit state functions $g_i(X) = 0 (i = 1, 2, 3)$ in Fig. 3. The individual failure mode probabilities in the first-order analysis are determined as

$$P_f = \phi(-\beta) \quad (21)$$

In standardized independent normal X space, the linear limit state function is given by

$$g_i(X) = \beta_i + \sum_{j=1}^n \alpha_{ij} x_j \quad (22)$$

where n is the number of random variables.

The angle between the two limit states provides information about the correlation of the two failure modes. The correlation coefficient is obtained as

$$\rho_{ij} = \sum_{r=1}^n \alpha_{ir} \alpha_{jr} = \cos \nu_{ij} \quad (23)$$

Once β_i , β_j and ρ_{ij} are obtained, the computation of joint probability of failure can be carried out. Eg. 19 and 20 can be used to compute the second-order bounds for system failure probability estimation.

The above method only provides a tool to approximately estimate the failure probability correlation of two different failure modes in a multi-disciplinary system. A more accurate approach would be the imposition of one failure mode on another mode and reanalysis of the latter. For example, consider two failure modes in a heat transfer system: structural failure and heat transfer failure. Structural failure happens when the stress, caused by the fluid pressure and temperature difference between outer and inner surfaces exceeds limiting value of strength. The heat transfer failure happens when the temperature of the contained liquid can not be kept at a certain level. When the thermal failure occurs, the increase in the temperature field also causes changes in stress field. The structural failure probability can be re-estimated under this changed stress field and the result can be considered as the interactive failure probability under influence of heat transfer failure. A numerical examples will be shown for this approach in a later chapter.





Symbol	Meaning of symbol
	Representation of an event
	Representation of an event of a failure
	OR-gate
	AND-gate

Figure 6: Standard symbols used in fault tree analysis

Probabilistic fault tree analysis

NESSUS system risk assessment (SRA) uses probabilistic fault tree analysis (PFTA). A fault tree is a mathematical construction of assumed component failure modes (bottom events) linked in series or parallel leading to a top event, which denotes system failure. Standard graphical symbols are used to construct the fault tree picture, by describing events and logical connections. These are shown in Fig. 6, and a simple PFTA is shown in Fig. 7.

- Fault Trees with a Single AND-GATE

Consider the fault tree in Fig. 8. Here the top event occurs if and only if all the bottom events E_1, E_2, \dots, E_n occur simultaneously. A system with AND-GATE is very similar to a series system structure.

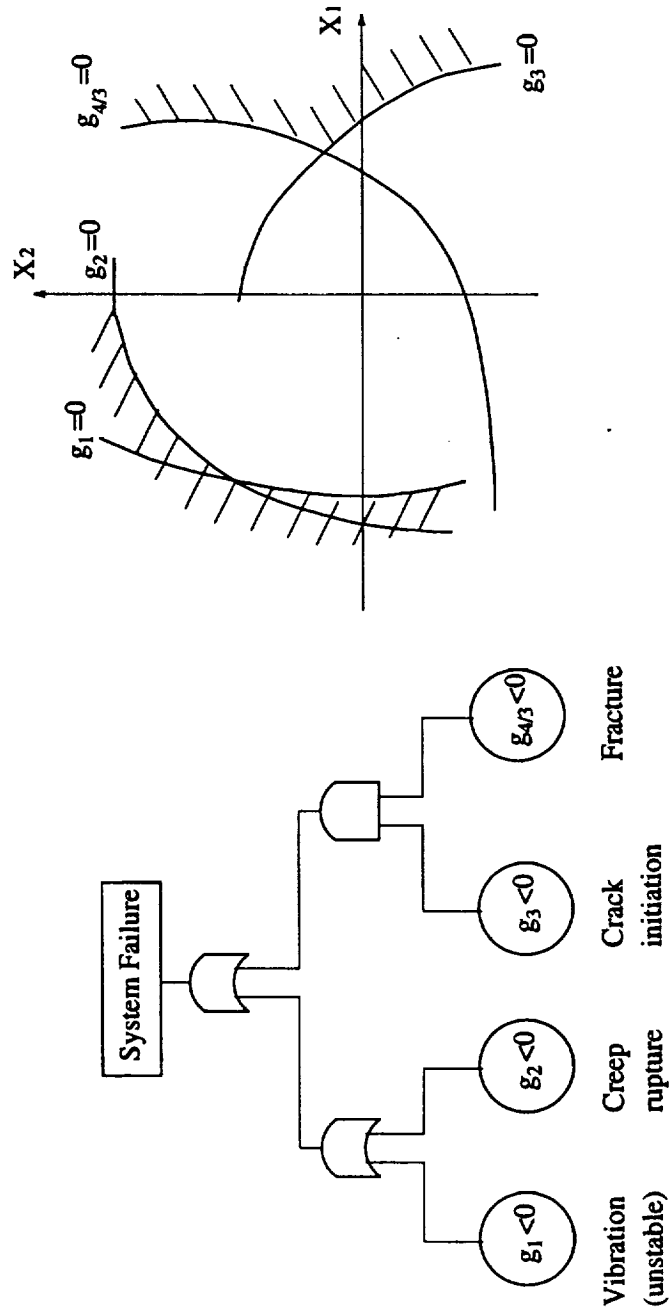


Figure 7: Probabilistic Fault Tree for System Reliability Example

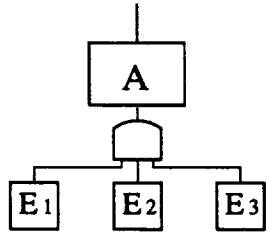
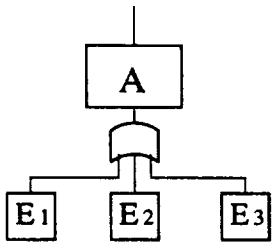
Symbols	Description
<p>AND-gate</p> 	<p>The AND-gate indicates that the output event A occurs only when all the input events E_i occur.</p>
<p>OR-gate</p> 	<p>The OR-gate indicates that the output event A occurs if any of the input events E_i occur.</p>

Figure 8: Fault tree with a single AND-GATE and a single OR-gate

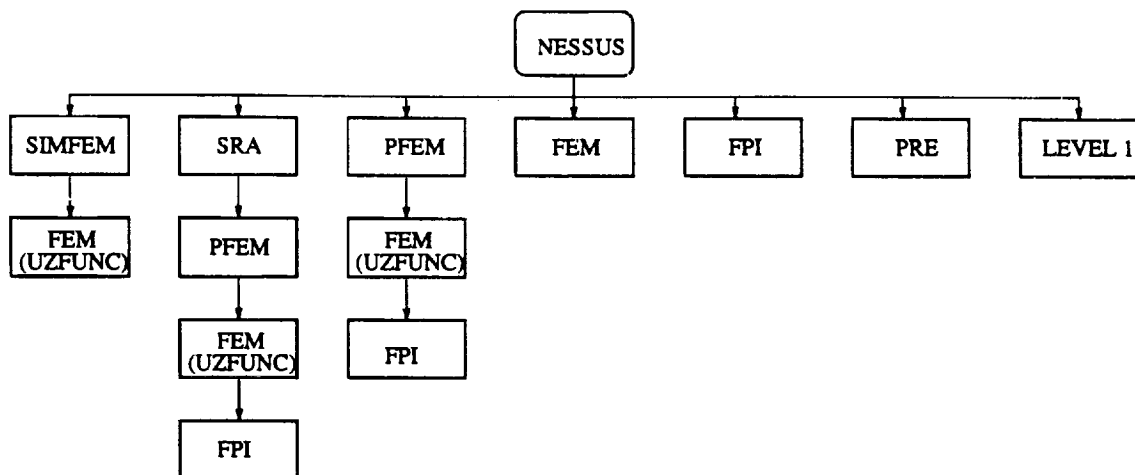


Figure 9: Schematic of NESSUS

- Fault Tree with a Single OR-GATE

Consider the fault tree in Fig. 8. The top event occurs if at least one of the bottom events E_1, E_2, \dots, E_n occurs. The structure of this fault tree is similar to the parallel system structure.

A schematic of Version 6.0 of the NESSUS (Numerical Evaluation of Stochastic Structures Under Stress) probabilistic structural analysis computer program is shown in Fig. 9. As shown in the diagram, the the NESSUS includes other modules, namely the System Risk Assessment (SRA) and Simulation Finite Element (SIM-FEM) modules. The random field pre-processor (PRE) provides data manipulation needed to express the uncertainties in a random field as a set of uncorrelated random variables. The user-subroutine which defines the response model (UZFUNC) enables

users to define required limit state with the computed response. This study will mainly use FEM, PFEM and FPI for reliability analysis. The NESSUS program is quite comprehensive with respect to structural reliability estimation. As mentioned in Chapter I, the purpose of this study is to develop a technique by which the NESSUS program can be used for the system reliability analysis of multi-disciplinary systems. The following chapters describe this technique in detail.

CHAPTER III

ANALOGY BETWEEN ENGINEERING SYSTEMS

Introduction

Since NESSUS/FEM program has been mostly applied only to structural analysis, a thermal or a fluid mechanical system needs to be converted through an analogous model to a structural system on which the NESSUS program can be applied for analysis. Then the probability analysis for a heat transfer system or a fluid mechanics system can be carried out by NESSUS. By doing so, a system with heat transfer, fluid flow and mechanical stress problems can be analyzed by NESSUS automatically with FEM, PFEM, FPI and SRA modules for system reliability analysis.

In this chapter, a new methodology is presented for one-dimensional steady-state heat transfer analysis and one-dimensional steady-state uniform flow problem using a structural finite element program. First, the use of the analogous models is introduced for the analysis of systems involving one-dimensional steady-state heat transfer and simple one-dimensional steady-state uniform flow in closed conduit systems.

Heat transfer analysis through structural analogy

One-dimensional steady-state heat transfer

We begin our analysis of one-dimensional, steady-state conduction by discussing heat transfer with no internal generation. The objective is to determine the expressions for temperature distribution and heat transfer rate in common geometries.

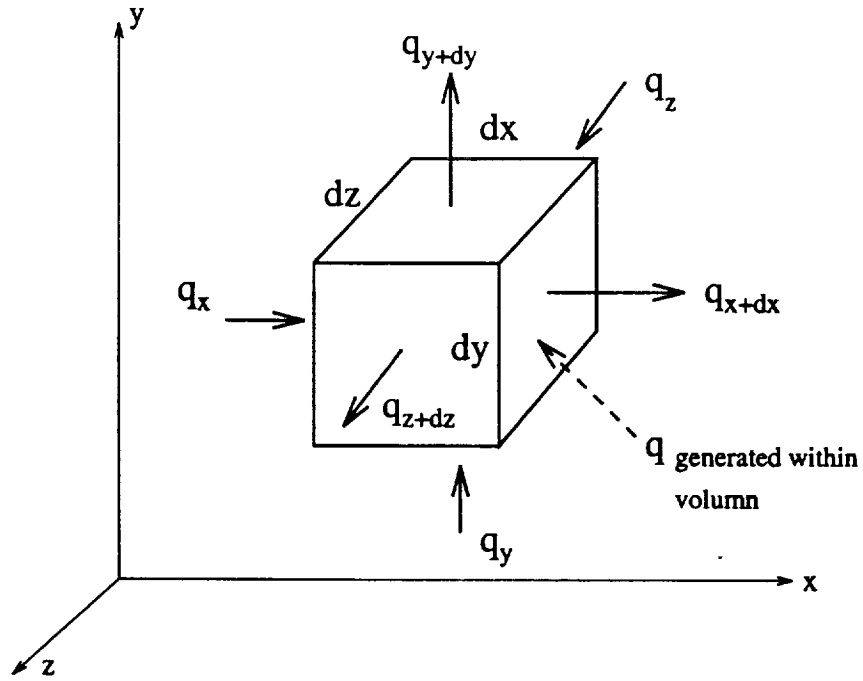


Figure 10: Differential volume for the derivation of the general equation of heat conduction

The concept of thermal conductivity (analogous to stiffness in stress analysis) is introduced as an aid to solving conduction heat transfer problems. Consider a three dimensional differential volume shown in Fig. 10, The general heat equation is

$$\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(K\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t} \quad (24)$$

where K is the thermal conductivity of the material. $K\frac{\partial T}{\partial x}$, $K\frac{\partial T}{\partial y}$, $K\frac{\partial T}{\partial z}$ are related to heat flux in a direction perpendicular to the surface. \dot{q} is the rate at which energy is generated per unit volume of the medium. The density ρ and specific heat c_p are two thermodynamic properties. The product ρc_p is the volumetric heat capacity. $\rho c_p \frac{\partial T}{\partial t}$ is the time rate of change of the internal (thermal) energy of the medium per unit volume.

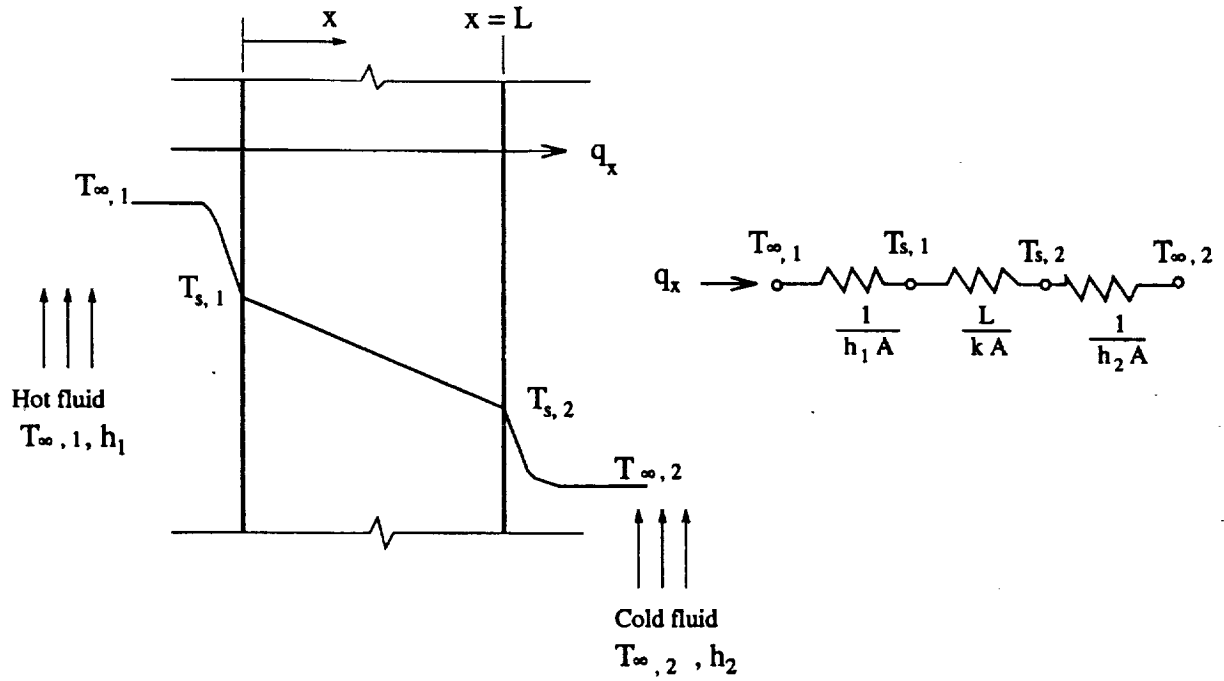


Figure 11: Heat transfer through a plane wall

If the heat transfer is one-dimensional and steady state, any differentiation with respect to time is equal to zero and there is no internal heat generation, so Eq. 24 reduces to

$$\frac{d}{dx} \left(K A \frac{dT}{dx} \right) = 0 \quad (25)$$

The heat flux is a constant, independent of x .

As shown in Fig. 11, a plane wall separates two fluids of different temperatures. Heat transfer occurs by convection from the hot fluid at $T_{\infty,1}$ to one surface of the wall at $T_{s,1}$, by conduction through the wall, and by convection from the other surface of the wall at $T_{s,2}$ to the cold fluid at $T_{\infty,2}$.

Assuming the thermal conductivity of the material to be constant, Eq. 25 may

be integrated twice to obtain the general solution

$$T(x) = C_1x + C_2 \quad (26)$$

To obtain the constants of integration, C_1 and C_2 , boundary conditions must be introduced. These are:

$$T(0) = T_{s,1} \quad (27)$$

$$T(L) = T_{s,2} \quad (28)$$

Applying the condition at $x = 0$ to the general solution, it follows that

$$T_{s,1} = C_2 \quad (29)$$

Similarly, at $x = L$

$$T_{s,2} = C_1L + C_2 = C_1L + T_{s,1} \quad (30)$$

in which case

$$\frac{T_{s,2} - T_{s,1}}{L} = C_1 \quad (31)$$

Substituting into the general solution, the temperature distribution is then

$$T(x) = \begin{bmatrix} N_1 & N_2 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix}$$

where $N_1 = 1 - \frac{x}{L}$, $N_2 = \frac{x}{L}$

The heat flow can be determined by Fourier's law, that is

$$q = -KA \frac{dT}{dx} \quad (32)$$

or

$$q = -KA \left[\frac{-(T_1 - T_2)}{L} \right] = \frac{KA}{L} (T_1 - T_2) \quad (33)$$

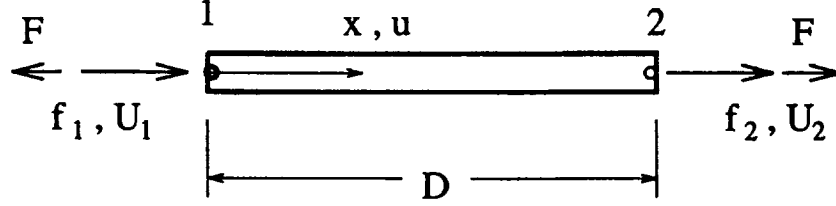


Figure 12: Bar subject to tensile force F

Stress analysis of a bar element

Now consider a linear-elastic, constant cross-sectional area (prismatic) bar element shown in Fig. 12. Using Hooke's law, the differential equation governing the linear-elastic bar behavior is

$$\frac{d}{dx} \left(ES \frac{dU}{dx} \right) = 0 \quad (34)$$

where U is the axial displacement function in the x direction and S and E are cross-sectional area and Young's modulus of elasticity respectively.

$$U = \begin{bmatrix} N_1 & N_2 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix}$$

where $N_1 = 1 - \frac{x}{D}$, and $N_2 = \frac{x}{D}$.

The strain-displacement relationship is

$$\varepsilon_x = \frac{dU}{dx} = \frac{U_2 - U_1}{D} \quad (35)$$

Table 1: Analogous quantities for structural and thermal systems

Heat transfer	Structure
Heat flux q	Nodal force f_1
Temperature $T(x)$	Displacement $U(x)$
Inverse of heat transfer resistance Conduction: $\frac{KA}{L}$, convection: hA	Structural stiffness $\frac{ES}{D}$

We obtain

$$F = ES \left(\frac{U_2 - U_1}{D} \right) \quad (36)$$

Also, by the nodal force sign convention of Fig. 12,

$$f_1 = -F \quad (37)$$

So Eq. 36 becomes

$$f_1 = \frac{ES}{D}(U_1 - U_2) \quad (38)$$

Analogous modeling between heat transfer and structure

Comparing Eq. 38 with Eq. 33, the similarities become apparent. These two equations indicate a direct analogy between heat transfer and structural analysis. The analogous quantities are listed in the Table 1.

With this analogy, we are able to model a heat transfer problem into a stress analysis problem.

In the plane wall, we refer heat transfer resistance of conduction R to $\frac{L}{KA}$, that is

$$R_{cond} = \frac{T_{s,1} - T_{s,2}}{q} = \frac{L}{KA} \quad (39)$$

Considering the structural system, Hooke's law provides stiffness of the form

$$k = \frac{f_1}{U_1 - U_2} = \frac{ES}{D} \quad (40)$$

Comparing Eqns. 39 and 40, and considering $\frac{KA}{L}$ and $\frac{ES}{D}$ as analogous qualities, $\frac{1}{R_{cond}}$ can be traced to be analogous to K .

A heat transfer factor may also be associated with convection at a surface. From Newton's law of cooling,

$$q = hA(T_s - T_\infty) \quad (41)$$

where h is Planck's constant of convection heat transfer coefficient, T_s is the surface temperature and T_∞ is the ambient temperature.

The thermal resistance for convection is then

$$R_{conv} = \frac{T_s - T_\infty}{q} = \frac{1}{hA} \quad (42)$$

The equivalent thermal circuit for the plane wall with convection surface conditions is shown in Fig. 11. The heat transfer rate may be determined from separate consideration of each element in the network, that is,

$$q = h_1 A (T_{\infty,1} - T_{s,1}) = \frac{KA}{L} (T_{s,1} - T_{s,2}) = h_2 A (T_{s,2} - T_{\infty,2}) \quad (43)$$

In terms of the overall temperature difference, $T_{\infty,1} - T_{\infty,2}$, and the effective thermal resistance R_{eff} , the heat transfer rate may also be expressed as

$$q = \frac{T_{\infty,1} - T_{\infty,2}}{R_{eff}} \quad (44)$$

Because the conduction and convection resistance are in series and may be summed up, it follows that

$$R_{eff} = \frac{1}{h_1 A} + \frac{L}{KA} + \frac{1}{h_2 A} \quad (45)$$

Consider a bar consisting of three different materials which are denoted as elements 1,2 and 3. The effective stiffness for this composite bar is

$$k_{eff} = \frac{1}{\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}} \quad (46)$$

Comparing the above equations Eq. 45 and Eq. 46, the analogy is $k_{eff} \longleftrightarrow \frac{1}{R_{eff}}$, that is

$$R_{eff} \longleftrightarrow \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} \quad (47)$$

Substituting with Eq. 45, we obtain

$$\frac{1}{h_1 A} + \frac{L}{KA} + \frac{1}{h_2 A} \longleftrightarrow \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} \quad (48)$$

where

$$k_1 = \frac{E_1 S_1}{D_1} \quad (49)$$

$$k_2 = \frac{E_2 S_1}{D_2} \quad (50)$$

$$k_3 = \frac{E_3 S_1}{D_3} \quad (51)$$

Substituting the corresponding terms in Eq.48, we obtain the equivalent quantities

$$E_1 \longleftrightarrow (h_1 A) \left(\frac{D_1}{S_1} \right) \quad (52)$$

$$E_2 \longleftrightarrow \left(\frac{KA}{L} \right) \left(\frac{D_2}{S_2} \right) \quad (53)$$

$$E_3 \longleftrightarrow (h_2 A) \left(\frac{D_3}{S_3} \right) \quad (54)$$

With these analogous quantities, we use the NESSUS/FEM beam element with the E values replaced by the values involving heat transfer problem outlined above. The boundary conditions for the bar are the end displacements corresponding to the ambient temperature of the wall. After the structural analysis, we get the temperature distribution from the corresponding displacement distribution in the output.

Heat transfer in composite walls

Equivalence concepts for thermal-structural analysis may also be used for more complex systems, such as composite walls and radial heat transfer systems. Fig. 13 shows a series composite wall. The one-dimensional heat transfer rate for this system can be expressed as

$$q = \frac{T_{\infty,1} - T_{\infty,4}}{\sum R} \quad (55)$$

where $T_{\infty,1} - T_{\infty,4}$ is the overall temperature difference and the summation includes all thermal resistances. Hence,

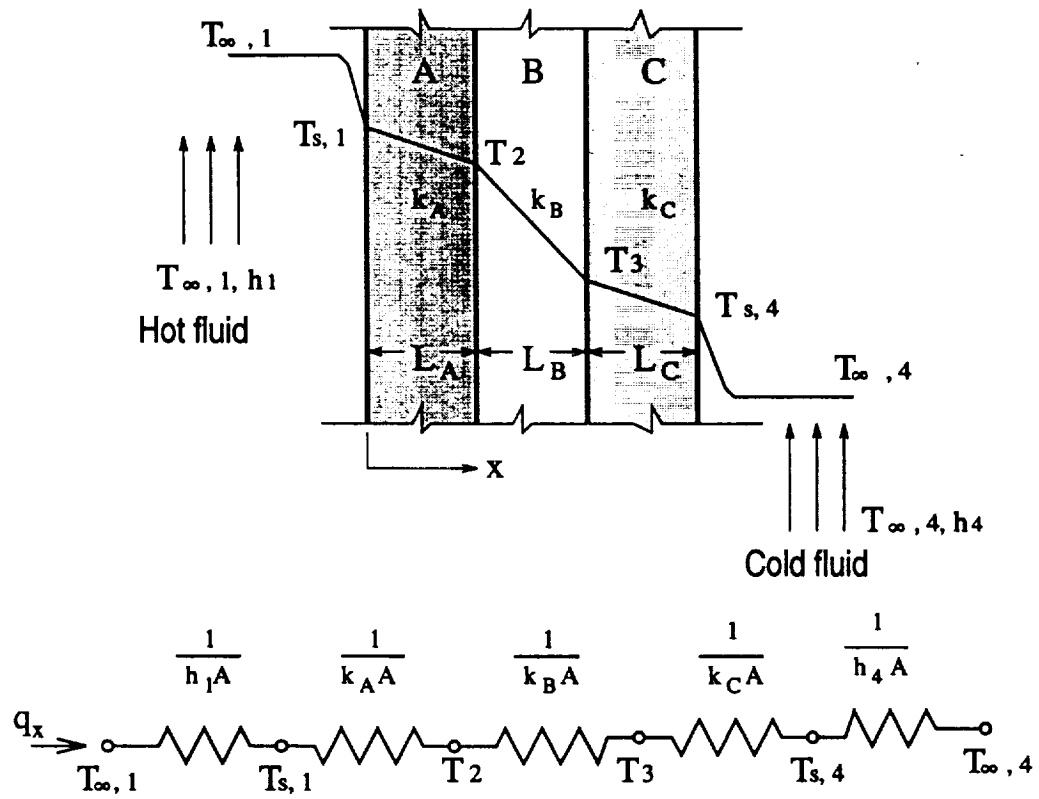


Figure 13: Equivalent thermal circuit of a series composite wall

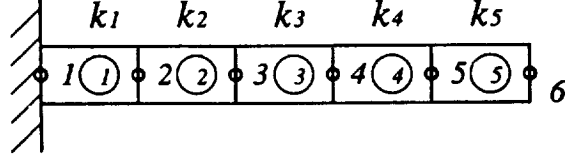


Figure 14: Structural analog for the series composite wall heat transfer

$$q = \frac{T_{\infty,1} - T_{\infty,4}}{R_1 + R_2 + R_3 + R_4 + R_5} \quad (56)$$

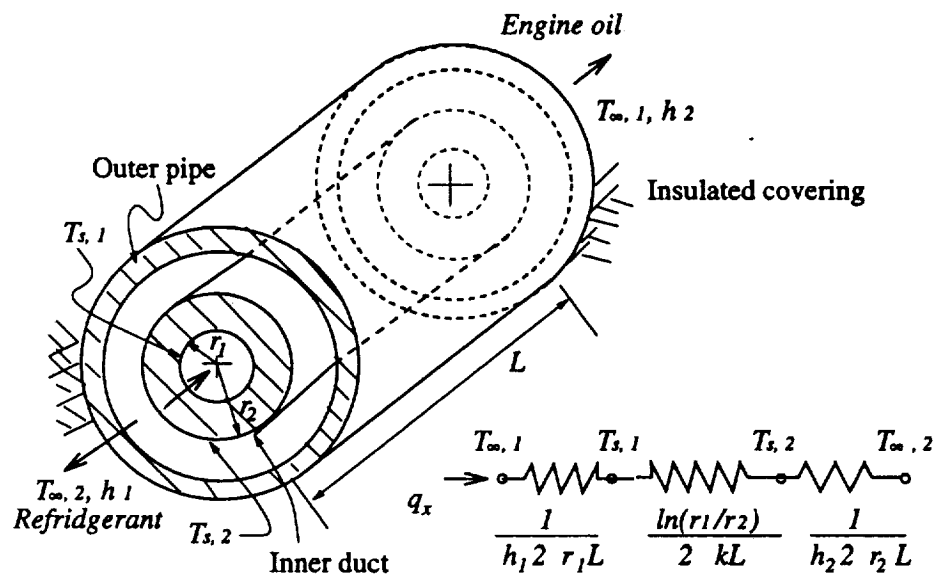
$$= \frac{T_{\infty,1} - T_{\infty,4}}{(1/h_1A) + (L_A/K_AA) + (L_B/K_BA) + (L_B/K_BA) + (1/h_4A)} \quad (57)$$

Alternatively, the heat transfer rate can be related to the temperature difference and resistance associated with each element. For example,

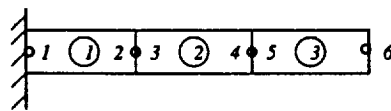
$$q = \frac{T_{\infty,1} - T_{s,1}}{(1/h_1A)} = \frac{T_{s,1} - T_2}{(L_A/K_AA)} = \frac{T_2 - T_3}{(L_B/K_BA)} = \dots \quad (58)$$

The analogous structural model for this series composite wall heat transfer problem is shown in Fig. 14. The bar consists of five elements with stiffnesses of k_1, k_2, k_3, k_4, k_5 . Using the mechanical structure equivalence for convection and conduction, we obtain

$$E = \left\{ \begin{array}{ll} (h_1A) \left(\frac{D}{S} \right) & \text{for convection} \\ \left(\frac{KA}{L} \right) \left(\frac{D}{S} \right) & \text{for conduction} \end{array} \right\}$$



(a) Heat exchanger



(b) Structural model

Figure 15: A heat exchanger for engine oil and refrigerant fluid

Heat transfer in radial systems

Cylindrical and spherical systems often experience temperature gradients in the radial direction only and may therefore be treated as one dimensional.

Fig. 15 shows an example of a heat exchanger, whose inner cylinder is used to store engine oil and the outer cylinder is used to transfer the refrigerant fluid to cool down the oil temperature. The outer insulated covering is assumed to isolate the system from the ambient environment. For steady state conditions with no heat generation, the appropriate form of the heat equation is

$$\frac{1}{r} \frac{d}{dr} \left(Kr \frac{dT}{dr} \right) = 0 \quad (59)$$

The rate at which energy is conducted across any cylindrical surface in the solid may be expressed as

$$q = -KA \frac{dT}{dr} = -K(2\pi rL) \frac{dT}{dr} \quad (60)$$

where $A = 2\pi rL$ is the area normal to the direction of heat transfer.

The thermal resistance is

$$R_{eff} = \frac{1}{h_1 2\pi r_1 L} + \frac{\ln(r_2/r_1)}{2\pi KL} + \frac{1}{h_2 2\pi r_2 L} \quad (61)$$

which includes both conduction and convection.

The heat transfer rate for a unit length of the cylinder therefore is

$$q = \frac{T_{\infty,1} - T_{\infty,2}}{\frac{1}{h_1 2\pi r_1 L} + \frac{\ln(r_2/r_1)}{2\pi KL} + \frac{1}{h_2 2\pi r_2 L}} \quad (62)$$

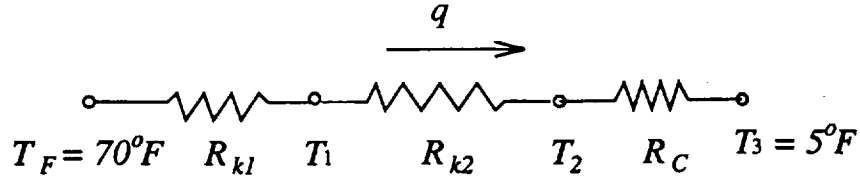
Table 2: Analogous quantities for heat transfer in a radial system.

Structure	Heat transfer
E_1	$(2\pi r_1 h_1) \left(\frac{D_1}{S_1} \right)$
E_2	$\left(\frac{2\pi K}{\ln(r_2/r_1)} \right) \left(\frac{D_2}{S_2} \right)$
E_3	$(2\pi r_2 h_2) \left(\frac{D_3}{S_3} \right)$

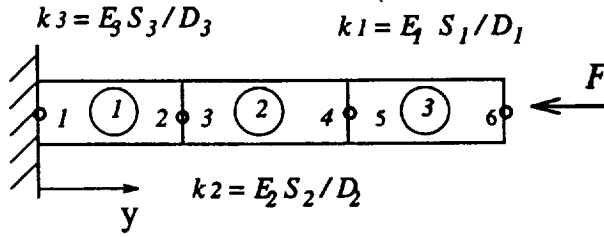
The structural analog for the cylinder is shown in Fig. 15(b). In this case, the mechanical structure equivalence is $k_{eff} \longleftrightarrow \frac{1}{R_{eff}}$. The corresponding equivalent quantities are listed in Table 2. The E values are input to NESSUS structural analysis, and the output displacements from NESSUS give the temperature distribution.

Numerical example for heat transfer solved with NESSUS/FEM

Fig. 17 shows the sectional view of the cylindrical copper heat exchanger which the engine oil flows through. The copper wall thickness is 0.281 in. The radius to the surface of the insulation pipe covering ($k_i = 0.428 Btu/(h - ft^2 - ^\circ F)$) is 1.33 in. The fluid in the outer container is controlled at a constant temperature of $70^\circ F$. The forced convection heat transfer occurs between the outer surface of the insulation covering and the flowing fluid with $h = 10 Btu/(h - ft^2 - ^\circ F)$. The surface temperature at the insulation covering is $35.298^\circ F$. The structural analogy model is used to determine the inside temperature of the tube, assuming steady state, one dimensional, uniform properties in each material, forced convection cooling and negligible thermal radiation. The conductivity coefficient of copper at room



(a) Thermal circuit



(b) Structural analogous model

Figure 16: Analogous model for the heat exchange

temperature is $K_c = 223 \text{ Btu}/(h - ft^2 - ^\circ F)$.

The thermal circuit is shown in Fig. 16(a). Fig. 16(b) shows the structural analog model for this heat transfer problem. Beam element type 98 in NESSUS/FEM element type library is adopted. Three elements represent three heat transfer forms involved in this problem, which are the forced convection between the surface of the insulation covering and the ambient air, conduction through the copper layer, and conduction through insulation covering, respectively. Therefore, in terms of the structural model, we must assign three different material elastic constants for this beam structure. Since the NESSUS/FEM utilizes the *Nodal-based data input*, two duplicate nodes are used at each boundary between elements 1 and 2, and between elements 2 and 3. The room temperature $70^\circ F$ becomes the boundary displacement

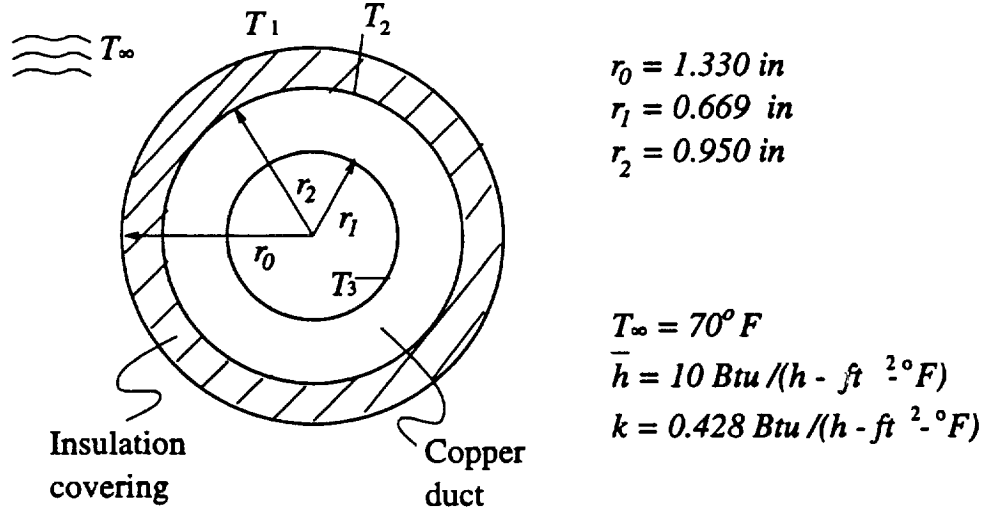


Figure 17: Section of a radial heat exchanger

70.0 at node 1. A concentrated load F at point 3 is -241.66 lb. The length of the structural element is 10.0 units, the sectional area is 1.0. It should be noted that the units used in the structural model here do not have any real meaning in terms of a real structure. They are simply used to facilitate the structural analysis.

The equivalent values are calculated as follows.

$$E_1 = \left(\frac{2\pi K_c}{\ln(r_2/r_1)} \right) \left(\frac{D_1}{S_1} \right) = \frac{2\pi \times 223}{\ln(0.95/0.669)} \times 10 = 39955.476 \quad (63)$$

$$E_2 = \left(\frac{2\pi K_i}{\ln(r_0/r_2)} \right) \left(\frac{D_2}{S_2} \right) = \frac{2\pi \times 0.428}{\ln(1.33/0.95)} \times 10 = 79.923 \quad (64)$$

$$E_3 = (2\pi r_0 h) \left(\frac{D_3}{S_3} \right) = 2\pi \times 11.33/12 \times 10.0 \times 10 = 69.63867 \quad (65)$$

$$F = 2\pi r_0 h (T_\infty - T_3) = 2\pi \times 1.33/12 \times 10(35.298 - 70) = -241.66 \quad (66)$$

NESSUS/FEM uses this data, and gives the output of the displacement distribution in the structure as follows:

$$U_1 = 70.000$$

$$U_2 = 35.298$$

$$U_3 = 35.298$$

$$U_4 = 5.0614$$

$$U_5 = 5.0614$$

$$U_6 = 5.0009$$

Converting the above displacement information to the equivalent temperature distribution, we obtain:

$$T_\infty = 70.000^\circ F$$

$$T_1 = 35.298^\circ F$$

$$T_2 = 5.0614^\circ F$$

$$T_3 = 5.0009^\circ F$$

The data and the output files are shown in Appendix A.

Fluid flow analysis through structural analogy

Equation of motion for fluid flow

The Bernoulli equation gives a relationship between pressure, velocity, and position or elevation in a flow field. Normally, these properties vary considerably in the flow, and the relationship between them if written in differential form is quite complex. The equation can be solved exactly only under very special conditions. There-

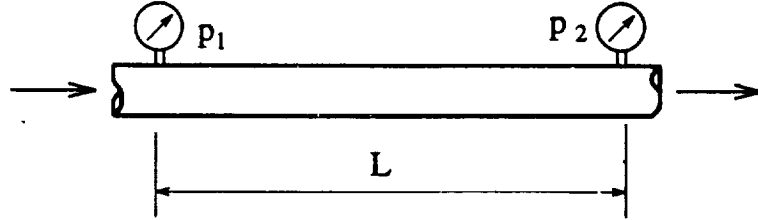


Figure 18: Fluid in a constant diameter duct

fore, in most practical problems, it is often more convenient to make assumptions to simplify the descriptive equations. The Bernoulli equation for steady, incompressible flow along a streamline with no friction (no viscous effects) is written as [10]

$$\frac{p}{\rho} + \frac{V^2}{2} + gz = C \quad (67)$$

where

p is fluid pressure

ρ is the density of the fluid

V is the flow velocity

$g = 32.174 \text{ ft/s}^2$, and z is height.

For a horizontal pipe shown in Fig. 18, $z_1 = z_2$. From continuity, $A_1 V_1 = A_2 V_2$. Because $D_1 = D_2$, then $A_1 = A_2$, and therefore, $V_1 = V_2$. The Bernoulli's equation reduces to

$$p_1 = p_2 \quad (68)$$

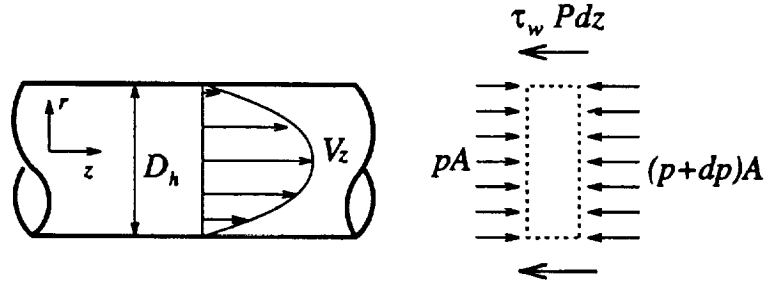


Figure 19: Control volume of a system: flow in a duct.

This result is not a proper description of the situation, however. For flow to be maintained in the direction indicated in Fig. 18, p_1 must be greater than p_2 in an amount sufficient to overcome friction between the fluid and the pipe wall. In order to apply Bernoulli's equation and obtain an accurate description, we must modify the equation with a friction term.

Consider flow in a pipe as shown in Fig. 19. A control volume that extends to the wall (where the friction force acts) is selected for analysis.

Note that a circular cross section is illustrated, but the results are general until we substitute specific equations for the geometry of the cross section. The forces acting on the control volume are pressure normal to the surface and shear stress acting at the wall. The momentum equation is [10]

$$\sum F_z = \int \int V_z \rho V_n dA \quad (69)$$

where

V_z is the fluid velocity along the longitudinal direction

V_n is the normal fluid velocity

Since the flow out of the control volume equals the flow in, the right-hand side of this equation is zero. The sum of the forces is

$$pA - \tau_w Pdz - (p + dp)A = 0 \quad (70)$$

where

A = cross-sectional area

Pdz = the surface area (perimeter times length) over which the wall shear τ_w acts

The equation reduces to

$$\tau_w Pdz + Adp = 0 \quad (71)$$

Rearranging and solving for pressure drop, we get

$$\frac{dp}{dz} = -\frac{4\tau_w}{D_h} \quad (72)$$

We have thus expressed the pressure drop per unit length of the conduit in terms of the wall shear and the hydraulic diameter. Eq. 72 is a general expression for any cross section. It is convenient to introduce a friction factor f , which is customarily defined as the ratio of friction forces to inertia forces:

$$f = \frac{4\tau_w}{\frac{1}{2}\rho V^2} \quad (73)$$

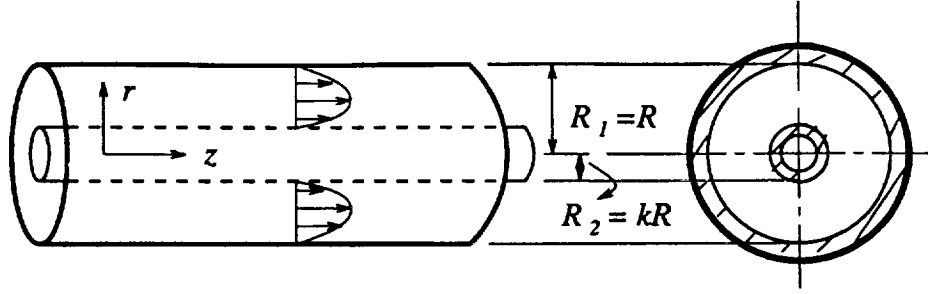


Figure 20: Laminar flow in an annulus

where V is the average flow velocity.

By substitution into Eq. 72, we obtain

$$dp = -\frac{\rho V^2}{2} \frac{f dz}{D_h} \quad (74)$$

Integrating this expression from point 1 to point 2 a distance L apart in the conduit yields

$$V^2 = \frac{2D_h}{\rho f L} \Delta p \quad (75)$$

Eq. 75 gives the relationship between the velocity and the pressure drop in the duct due to friction. This equation can be applied to two flow regimes - laminar and turbulent flow. However, caution must be exercised when determining the friction factor f .

This equation can also be applied to flow through noncircular cross section such as rectangular duct and annulus. Fig. 20 shows the laminar flow in an annulus.

Table 3: Analogous quantities between structural and flow systems

Fluid mechanics	Structure
Square of velocity V^2	Nodal force $f_1 = -F$
Pressure distribution $p(x)$	Displacement $U(x)$
Flow factor $\frac{2D_h}{\rho f L}$	Structural stiffness $\frac{ES}{D}$

The annulus flow area is bounded by the inside surface of the outer duct (radius R_1) and the outside surface of the inner duct (R_2). We define the ratio of these diameters as

$$k = \frac{R_2}{R_1} \quad (76)$$

in which $0 < k < 1$.

The friction factor used in Eq. 75 is defined as [10]

$$\frac{1}{f} = \frac{R_e}{64} \left[\frac{1+k^2}{1-k} + \frac{1+k}{\ln(k)} \right] \quad (77)$$

where $R_e = \frac{\rho V(2R)}{\mu}(1-k)$.

Compare Eq. 75 with Eq. 38 concerning the beam structure subjected to the end nodal force, as discussed in previous section:

$$f_1 = \frac{ES}{D}(U_1 - U_2) \quad (78)$$

We are now able to set up the analogous quantities listed in Table 3.

Numerical example of flow in a tube solved with NESSUS/FEM

Consider the refrigerant flow in a copper tube as an example to demonstrate how NESSUS/FEM can be applied to problems in fluid mechanics.

A horizontal copper duct as shown in Fig. 18 with inside radius of 0.669 *in*, and 1,200 *in* in length. If the inflow pressure p_1 is 1838.7 *psi*, assuming the refrigerant is Freon F-12 under a temperature of 5° *F*, ρ is 0.0499 *lb/in*³. The friction factor f is assumed to be 0.03, and V is 15.5 *in/sec*. The objective is to obtain the outflow pressure p_2 using NESSUS/FEM.

First of all, we need to identify all the equivalent quantities for structural analysis. We assume a single element beam structure subjected to a concentrated force equal to -240.25 units. The beam element has a section of 0.1 *in* \times 0.1, and a length of 1.0. The boundary condition is an initial displacement of 1838.3 units at node 1. Again, it should be mentioned that the units used here do not have real meaning in terms of a real structure. According to Table 3, the analogous quantities can be obtained as follows

$$E = \left(\frac{2D_h}{\rho f L} \right) \left(\frac{D}{S} \right) = \frac{2 \times 2 \times 0.669}{0.0499 \times 0.03 \times 1200} \times 10 = 14.896$$

$$f_1 = V^2 = 15.5^2 = 240.25$$

$$F = -240.25$$

This data is input to NESSUS/FEM, and the displacement at point 2 is obtained as 1677.4. Converting this displacement to the fluid model, we get the output pressure $p_2 = 1677.4$ *psi*.

The data and the output files are shown in Appendix B.

CHAPTER IV

MULTI-DISCIPLINARY SYSTEM RELIABILITY ANALYSIS

Introduction

After transforming the heat transfer and fluid mechanics problems into corresponding structural analog models and using NESSUS/FEM to perform the finite element analysis, we can define the individual failure modes in NESSUS/FPI. Then NESSUS/PFEM can be employed to integrate FEM and FPI programs to obtain the failure probability and CDF for each failure mode. The failure mode for heat transfer problem would be defined as, for example, the event that the temperature at a certain location is lower or higher than the required temperature. The failure mode for fluid flow would be defined as the flow pressure exceeding a certain pressure level, and the structural failure is defined as the stress exceeding either the ultimate strength or the yield strength of the material.

Upon the completion of failure probability analyses of individual failure modes, the system failure analysis can be pursued. The different failure modes involved in a system have different impacts on the overall performance of a system. Some types of failure such as structural failure are critical to the system. If the material used to construct the main parts of the system fails, the whole system can no longer function. Such failure is called critical failure. Other failures modes such as thermal failure of a heat exchanger do not destroy the system but degrade the performance of the system.

Such failure is referred as to functional failure. The function of fluid flow will fail when the outflow pressure rises higher than the designed value, but the system can still be working until the pressure increases to the level which will cause the system to shut down. The individual failure modes can also be correlated to each other. For example, the temperature field in the thermal failure mode affects the stress field in the structural mode. The flow pressure definitely has impact on the stress. However, in some cases, the component-level events in a system is considered as independent events. In the example which will be discussed later in this chapter, the thermal failure and fluid flow failure modes do not share correlated input parameters, so they are considered as independent of each other.

Using the analogy method, the thermal and fluid flow problems are analyzed similar to the structural model by means of NESSUS. For physically correlated events, the failure mode of one event is imposed on the other. In a system consisting of structural, thermal and fluid flow modes, the thermal and flow failures are imposed into the structural failure analysis to study the impact of correlated events. The failure probability of the whole system is then estimated based on the output from the above analyses.

Individual failure analysis

Structural failure mode

For a copper duct of a heat exchanger shown in Fig.17 in Chapter III, the structural failure mode is defined as that when the tensile stress exceeds the yield strength $f_{yield} = 8.0$ ksi. In finite element modeling, we use Element type 153 in the NESSUS/FEM file. This element is a four-noded quadrilateral lying in the global

zr -plane which is defined by cylindrical coordinates.

This structure is subjected to two types of load - fluid pressure from the inside flow and the stress caused by temperature difference between the outer surface and the inner surface.

One convenient feature of the NESSUS/PFEM is that we can impose different temperatures at the inner surface of the pipe and obtain the different probability results under different temperature conditions. This enables us to investigate the effect of different temperature levels on the structural failure probability. By doing so, the relationship between the failure modes in two disciplines - heat transfer and structural mechanics - is established. This is a significant step toward the system reliability analysis with physically correlated failure modes. This will be demonstrated in a later section.

For this structural failure model, we first suppose that the temperature failure (which will be described in the following section) did not occur, that is, the temperature at the inner surface of the duct is below 5.0009°F . Given that the inner surface temperature is 4.0000°F , using the FEM we obtain the outer surface temperature as 4.0605°F .

Also, we assume that the outlet flow pressure is under 1677.4 psi which enables the system to work properly. We assume that the outlet flow pressure is 1577.4 psi.

We input this temperature and flow pressure profile in the structure FEM data file, with the random variables defined in Table 4.

Table 4: Random variables for structural model

Variable	Mean value	Distribution	C.O.V.
Pressure P	1577.4 psi	Normal	0.2
Modulus of elasticity E	1.7×10^7 psi	Normal	0.1
Coefficient of thermal expansion α	9.5×10^{-6}	Normal	0.1

Table 5: CDF corresponding to different tensile strength levels

Z-level (strength)	CDF	Z-level (strength)	CDF
-206.89941 psi	0.00000017	4385.4964 psi	0.81593991
558.49989 psi	0.00002067	5150.8957 psi	0.97128351
1323.8992 psi	0.00096767	5916.2950 psi	0.99813412
2089.2985 psi	0.01786435	6681.6943 psi	0.99995188
2854.6978 psi	0.13566610	7447.0935 psi	0.99999952

The result attached in Appendix C indicates that the structural reliability when the heat exchanger is working properly in thermal and fluid aspects is 0.99999999. The failure probability is expressed as $1 - P_{reliability}$. Therefore, the structural failure probability is 1.0×10^{-9} .

The key word *response type* in the NESSUS input data file, FPI section, is set equal to 3 which means that the response quantity used in limit state function is stress. The corresponding keyword *analysis type* in FPI section is first set equal to 1 which means that the probability analysis is for a single Z-level. The Z-level in this case is 8,000 psi. The probability result will be under the condition of $\sigma < 8,000$ psi, i.e., the structural reliability of the system under certain thermal and fluid flow working conditions.

The CDF is obtained by using PFEM by setting *analysis type* in FPI section equal to 0 which automatically generates a set of different values of Z_0 (i.e., Z levels for a series of stress values) for probability analysis. The CDF values corresponding to different strength Z-levels are shown in Table 5.

The CDF chart is shown in Fig. 21. It should be noted that the first line of the data which contains negative Z-level is eliminated because negative stress is considered impractical in this model. The input and output files are attached in Appendix C as well.

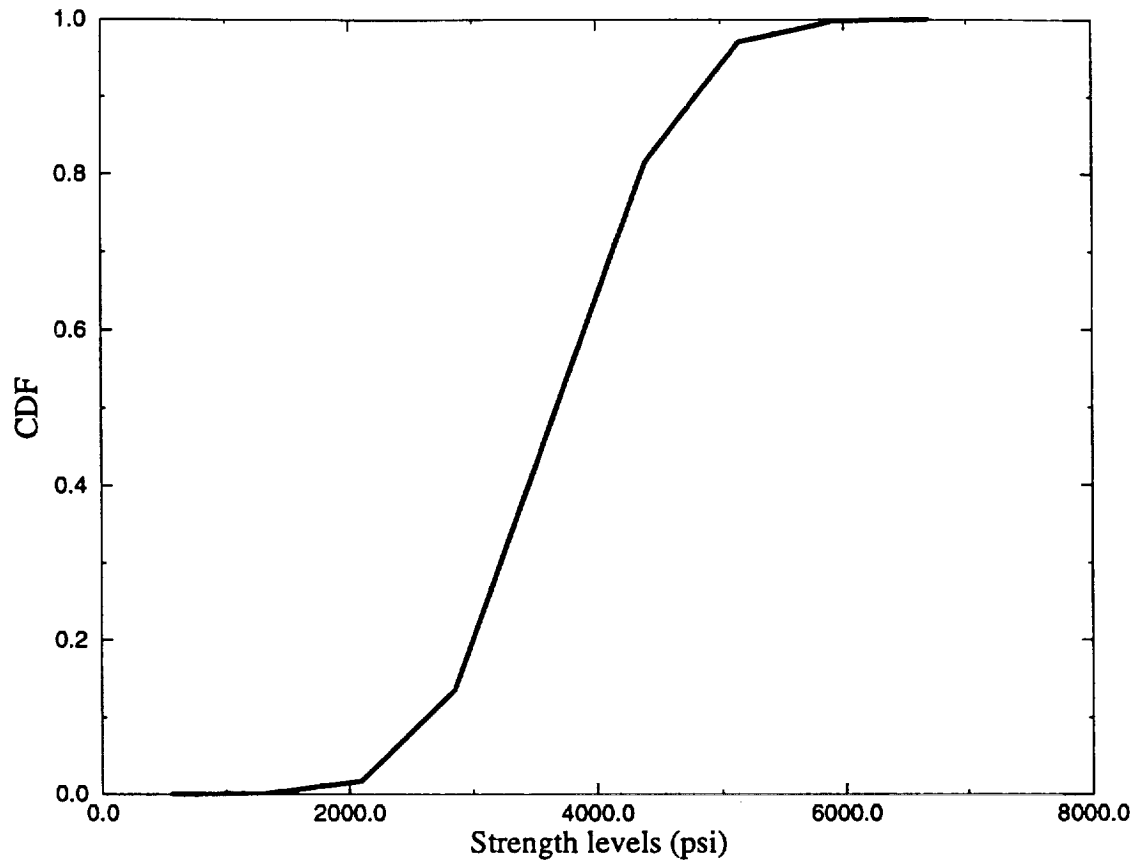


Figure 21: CDF of structural reliability of refrigerant duct

Table 6: Random variables for thermal model

	Mean value	Distribution	Coefficient of variation
K_c	223.0	Normal	0.1
K_i	0.428	Normal	0.1
h	11.3	Normal	0.1

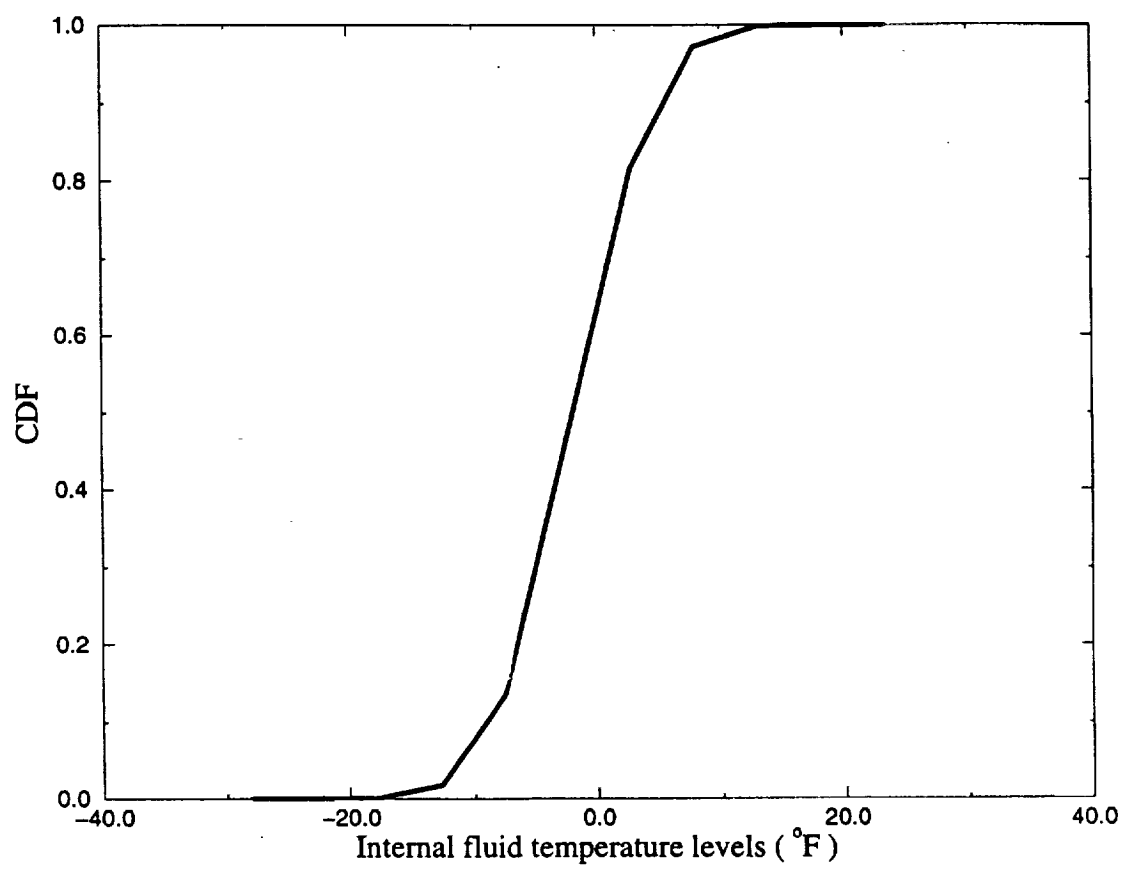


Figure 22: CDF of internal fluid temperature of refrigerant duct

Failure mode in heat transfer

Using the same example of a heat exchanger as in Fig. 17, we define a failure event when the inside temperature is higher than $5^{\circ}F$, because the refrigerant will not function properly beyond $5^{\circ}F$ which is considered as a failure in the device we studied. First the input data for NESSUS/PFEM is set up to obtain the reliability under this failure mode, then the data file is set up with different Z - levels to obtain the CDF, which provides reliability estimate corresponding to different temperature levels. The random variables K_c and K_i and h for heat transfer are defined in Table 6.

In order to use NESSUS/PFEM, the analogous quantities E_1 , E_2 , E_3 and F are calculated from Eqs. 63, 64, 65, 66. Because the distribution of the random variables K_c and K_i and h is normal and E_1 , E_2 , E_3 are linear to K_c and K_i and h , the distribution of random variables E_1 , E_2 , E_3 is also normal. The mean values and standard deviations of E_1 , E_2 and E_3 are input to NESSUS/PFEM.

The Z -level is $5.0^{\circ}F$, so $P(Z < Z_0)$ is the probability the device can keep the inside fluid temperature under $5.0^{\circ}F$, which is the thermal reliability of the system. We set up the keyword in FPI section *analysis type* equal to 1 which means the probability analysis is performed for a single Z -level. The result is attached in Appendix D. The thermal reliability of this device is 0.9099214. Therefore, the thermal failure probability is 9.00786×10^{-2} .

The CDF is obtained by setting up the FPI keyword *analysis type* equal to 0 which automatically generates a set of different values for Z_0 (i.e., Z -levels). The

Table 7: Random variables for flow model

	Mean value	Distribution	Coefficient of variation
D_h	0.669	Normal	0.1
V	18.71	Normal	0.1

CDF output is shown in Appendix D and the CDF curve is shown in Fig. 22.

Failure mode in fluid flow

Next we consider the one-dimensional fluid flow in a duct of a heat exchanger. The failure mode is defined as the pressure at a certain point along the duct rising above the value at which the system cannot function properly.

The example of a duct in a heat exchanger shown in Fig. 16 is used. The only difference is that V is assumed to be 18.7 *in/sec*. We define the failure mode when pressure rises above 1677.4 psi. The Z -level is therefore 1677.4 psi. The keyword *response type* is set as 1 for the displacement output which is the analogy of the pressure. The random variables related to fluid flow are defined in Table 7. The analogous quantities for use in NESSUS/PFEM are calculated according to Table 3 as follows:

$$E = \left(\frac{2D_h}{\rho f L} \right) \left(\frac{D}{S} \right) = \frac{2 \times 2 \times 0.669}{0.0499 \times 0.03 \times 1200} \times 10 = 14.896$$

$$f_1 = V^2 = 18.7^2 = 350.0$$

$$F = -f_1 = -350.0$$

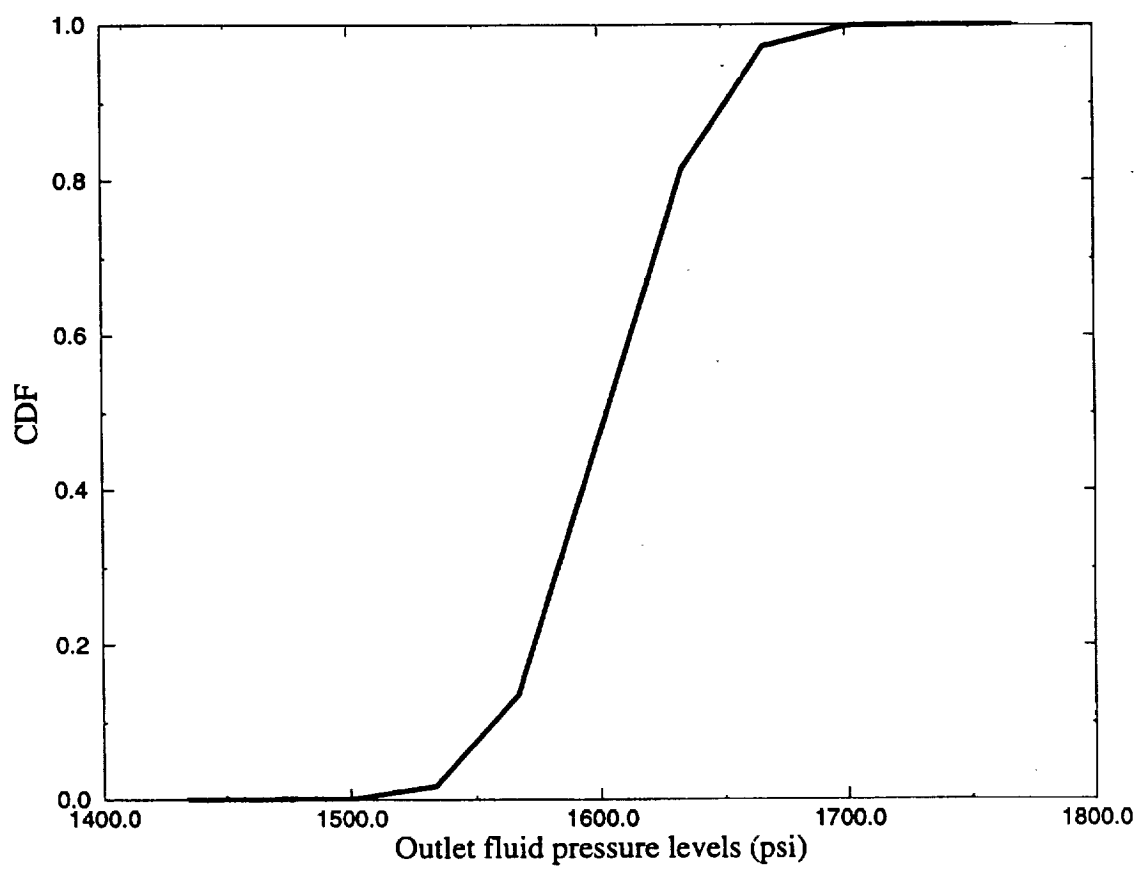


Figure 23: CDF of fluid pressure of refrigerant in the duct

Since E is a linear function of D_h , its distribution is also normal. But since $f_1 = V^2$, the distribution of f_1 is actually chi-square (χ^2). However, we have used the normal distribution for f_1 in this study as an approximation. The friction factor f is assumed to be a constant.

A NESSUS/PFEM input data file is compiled. The reliability is obtained by setting the keyword *analysis type* equal to 1, and the CDF is obtained by setting it to a value of 0 which automatically generates a set of different values for Z_0 . The PFEM input and output files are shown in Appendix E and the CDF curve is shown in Fig. 23.

The reliability is 0.98670241 and therefore the failure probability of the output flow pressure being higher than 1677.4 psi is 1.329759×10^{-2} .

Multi-disciplinary system reliability

After the individual failure modes are identified and analyzed, the system reliability analysis can be pursued. Fig. 24 shows a device which is used to transfer refrigerant fluid through a copper duct. The duct is installed in an enclosed chamber which is maintained at a constant temperature of $70^\circ F$. The thickness of the copper wall is 0.281 in. The radius to the surface of the insulation pipe covering (mean value of K_i equals to $0.428 \text{ Btu}/(h - ft^2 - ^\circ F)$, c.o.v. equals to 0.1) is 1.33 in. Forced convection heat transfer occurs with $h = 10 \text{ Btu}/(h - ft^2 - ^\circ F)$ (mean value with c.o.v. equals to 0.1). The thermal conductivity of copper K_c has a mean value of $233.0 \text{ Btu}/(h - ft^2 - ^\circ F)$ with a c.o.v. 0.1. The surface temperature at the insulation covering is $35.298^\circ F$. The inflow pressure p_1 is designed to be 1838.7

psi. The refrigerant is Dichlorodifluoromethane (Freon F-12) under a temperature of $4^{\circ}F$. ρ is 0.0499 lb/in^3 . The friction factor f is assumed to be 0.03, and V is 18.71 in/sec (assuming V^2 is normal distribution and has a c.o.v. of 0.1). The inner radius of the duct has a mean value of 0.669 in and a c.o.v. of 0.1.

The above data are the same as the model shown in Fig. 17 of Chapter III, which are used in FEM analysis and reliability estimation of individual failure modes. The system shown in Fig. 24 is simply the combination of the previous individual models which have been analyzed in different disciplines. The following is to demonstrate how the analysis results of the individual failure modes can be integrated into the analysis of a whole system.

The failure of the system consists of the individual failure modes in three disciplines: structural failure, thermal failure and fluid mechanics failure.

- First of all, the duct should work without any damage to the structure, i.e. the duct should be structurally sound without yied or crack. If yield occurs, then structural failure is assumed to occur. We denote the structural failure as E_1 . The structural failure is a critical failure in this system.
- The refrigerant liquid this device transports is sensitive to temperature changes. The requirement is that the temperature cannot be higher than $5^{\circ}F$ for the next process to proceed. If the temperature of the liquid rises higher than $5^{\circ}F$, then thermal failure occurs, which we refer to as E_2 . The thermal failure is a non-critical functional failure in this system.
- It is required that the fluid flow be maintained at a certain presure at the

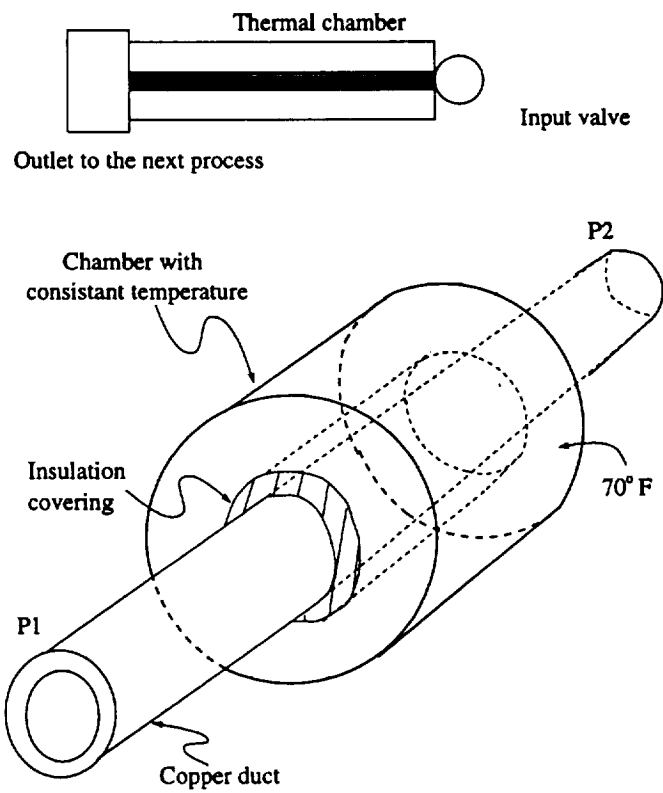


Figure 24: Refrigerant duct through a chamber

ends which enables the refrigerant fluid to maintain a steady speed to provide constant volume in the device. If the flow pressure rises higher than 1677.4 psi, the flow failure occurs, which is referred to as E_3 . The flow failure is a non-critical functional failure in this system.

From the previous section, the probability of the individual failure modes of the system shown in Fig. 24 have been obtained, which are:

$$P(E_1) = 1.0000000 \times 10^{-9}$$

$$P(E_2) = 9.0078600 \times 10^{-2}$$

$$P(E_3) = 1.3297590 \times 10^{-2}$$

As was indicated in the previous section, we can also impose one failure mode upon the other. In this case we can impose the thermal failure (which happens when the fluid temperature rises above $5^\circ F$) and the fluid pressure failure (which happens when the outlet fluid pressure rises above 1677.4 psi) upon the structure respectively. In the FEM file for the structure model, the corresponding data are modified to impose those failures.

First, we assume that the temperature failure occurs while the fluid pressure is still lower than 1677.4 psi, say 1577.4 psi, i.e., the fluid flow is operation in safe mode. By redefining the *temperature* profile in the FEM data deck as $6.06^\circ F$ (for example) in the inner layer of the wall and $6.12^\circ F$ in the outer layer of the wall, which mean the thermal failure occurs, we impose the thermal failure to the structural model. The PFEM result gives us the structural failure probability under the condition that the thermal failure occurs. In this case, the structural reliability is 0.99999996, therefore,

the failure probability $P(E_1/E_2) = 4.0 \times 10^{-8}$.

Now we impose the fluid pressure failure upon the structural model. The fluid mechanical failure occurs when the pressure at the outlet rises above 1677.4 psi, say 1777.4 *psi*. The structural FEM file is modified by redefining the pressure profile according to this failure pressure. It should be noted that the temperature profile should remain under the normal working condition, which is that the temperature in the inner layer of the wall of duct is under 5°F, say 4°F, i.e., the thermal aspect of the system is operating in the safe zone. The result indicates that in this case, the structural reliability is 0.99999633, therefore, the conditional probability, $P(E_1/E_3) = 3.67 \times 10^{-6}$.

Next, both thermal and fluid mechanical failures are imposed that is, the fluid temperature rises above 5°F, and the outlet flow pressure rises above 1677.4 psi. Modifying the input FEM data deck in structural PFEM file with inner surface temperature of 6.06°F, and the fluid pressure of 1777.4 *psi*, we can get the result of the structural reliability of 0.99999227, which means, $P(E_1/E_2E_3) = 7.73 \times 10^{-6}$.

The conditional probabilities of structural failure have been obtained as

$$P(E_1/E_2) = 4.00 \times 10^{-8}$$

$$P(E_1/E_3) = 3.67 \times 10^{-6}$$

$$P(E_1/E_2E_3) = 7.73 \times 10^{-6}$$

System reliability computation

System reliability analysis can be performed in two different ways, depending on the definition of the system failure. In the first (traditional) method, we define that

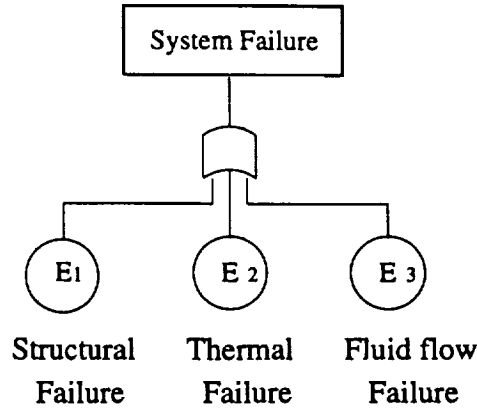


Figure 25: Fault tree for the system with three critical failures

the system failure occurs when any component-level failure occurs. In the system involving structural, thermal, fluid mechanical failure modes, i.e., E_1 , E_2 and E_3 , the system failure can be illustrated in a fault tree shown in Fig. 25.

As discussed in Chapter II, the probability of system failure $P(E)$ can be obtained using the following equations:

$$P(E) = P(E_1 \cup E_2 \cup E_3) \quad (79)$$

The above expression can be expanded as:

$$P(E) = P(E_1) + P(E_2) - P(E_1 \cap E_2) + P(E_3) - P(E_1 \cap E_3) - P(E_2 \cap E_3) + P(E_1 \cap E_2 \cap E_3) \quad (80)$$

Since the joint probability is not always available, an approximate method is to consider the individual failure modes as independent and ignore the correlations. In our case however, the conditional probabilities have been calculated. Therefore, the

system failure probability can be computed as

$$P(E) = P(E_1) + P(E_2) + P(E_3) - P(E_1/E_2)P(E_2) - P(E_1/E_3)P(E_3) - P(E_2/E_3)P(E_3) + P(E_1/E_2E_3)P(E_2E_3) \quad (81)$$

For systems involving many failure modes, approximation methods are used to predict the system failure probability (or reliability), such as first-order bounds or second-order bounds [1].

Since no correlation is assumed between the failures of flow pressure and fluid temperature, we assume $P(E_2 \cap E_3)$ is equal to $P(E_2)P(E_3)$ in our analysis.

Substituting the numerical results from the previous discussion into Eq. 81, we obtain the probability of the system failure of the heat exchanger, $P(E) = 0.10217832$.

As mentioned before, the above failure probability is an estimation of system failure in case any failure occurs which includes both critical and non-critical functional failures. Now we will pursue the probability estimation for the system critical failure which, in our case, is structural failure. During the service cycles, the thermal and fluid mechanical failures may be non-critical, i.e., their occurrence does not cause total system failure. They will cause the system to fail in some functions as designed, such as keeping the fluid under certain temperature or keeping the outlet fluid pressure under certain value. However, if the the system keeps operating, the changes in temperature and fluid pressure will cause progressive damage to the structure due to load redistribution. The estimation of critical structural failure of the system has to consider the progressive damage caused by all components. Structural reanalysis

Table 8: Structural failure probability under various temperatures

$T(^{\circ})$	0.0	5.0	10.0	15.0
$P(E_1)$	0.0	0.0	0.0	0.0

is used to account for the effect of non-critical damage on critical failure mode. In the refrigerant model, a reanalysis procedure is performed to accurately estimate the failure region segments for structural failure mode affected by progressive damage caused by thermal and fluid pressure changes within the system. The overall structural failure probability is obtained through the union of the failure region segments defined by each limit-state function.

We can also impose a series of temperatures under which the system may be operating upon the structural model to examine the temperature impact on the structural failure probability. Just as we did before, the failure probability is obtained as $(1 - \text{Reliability})$. In this case, we still assume inner fluid pressure is 1577.4 psi, which means that the fluid flow mode of the system is operating in the safe zone. The results are shown in Table 8.

Table 8 shows that when the fluid pressure is not considered as a random variable in perturbation for probability analysis, the temperature changes do not have a significant impact on structural reliability of the system.

We can also get the structural failure probability under different pressure conditions by defining a series of the pressure profiles in the FEM data deck for structural

Table 9: Structural failure probability under various pressures

Pressure (psi)	2700.0	2900.0	3000.0	3200.0	3300.0	3400.0	3420.0
$P(E_1)$	0.000	1.000×10^{-9}	1.920×10^{-6}	7.687×10^{-3}	9.195684×10^{-2}	4.06468×10^{-1}	4.92622×10^{-1}

model. The results are listed on Table 9. In this case, we assume that the inner temperature is 4°F.

It should be noted that the NESSUS/PFEM input file *gfun.dat* is different from the previous structural PFEM file in which the pressure is defined as a random variable. In the program [Ang and Tang, 1984] to calculate the union of the region segments, the random variables once defined can not be changed for different limit states. Since in the pressure profile the different pressure levels are presented, pressure should not be defined as a random variable. Therefore only two random variables are involved in *gfun.dat* - modulus of elasticity E and coefficient of thermal expansion α . The *gfun.dat* and various *gfun.mov* files are shown in Appendix F.

There are two ways of quantifying the effect of progressive damage on critical failure. The first is simply to compute the variation of critical failure probability with respect to progressive damage. This is shown in Fig. 26 for various pressure levels.

An alternate way is to compute the progressive damage on overall critical failure probability. If each critical failure limit state segment for each progressive damage gives the event E_i , then the overall critical failure probability is $P(\cup_{i=1}^n E_i)$, where n

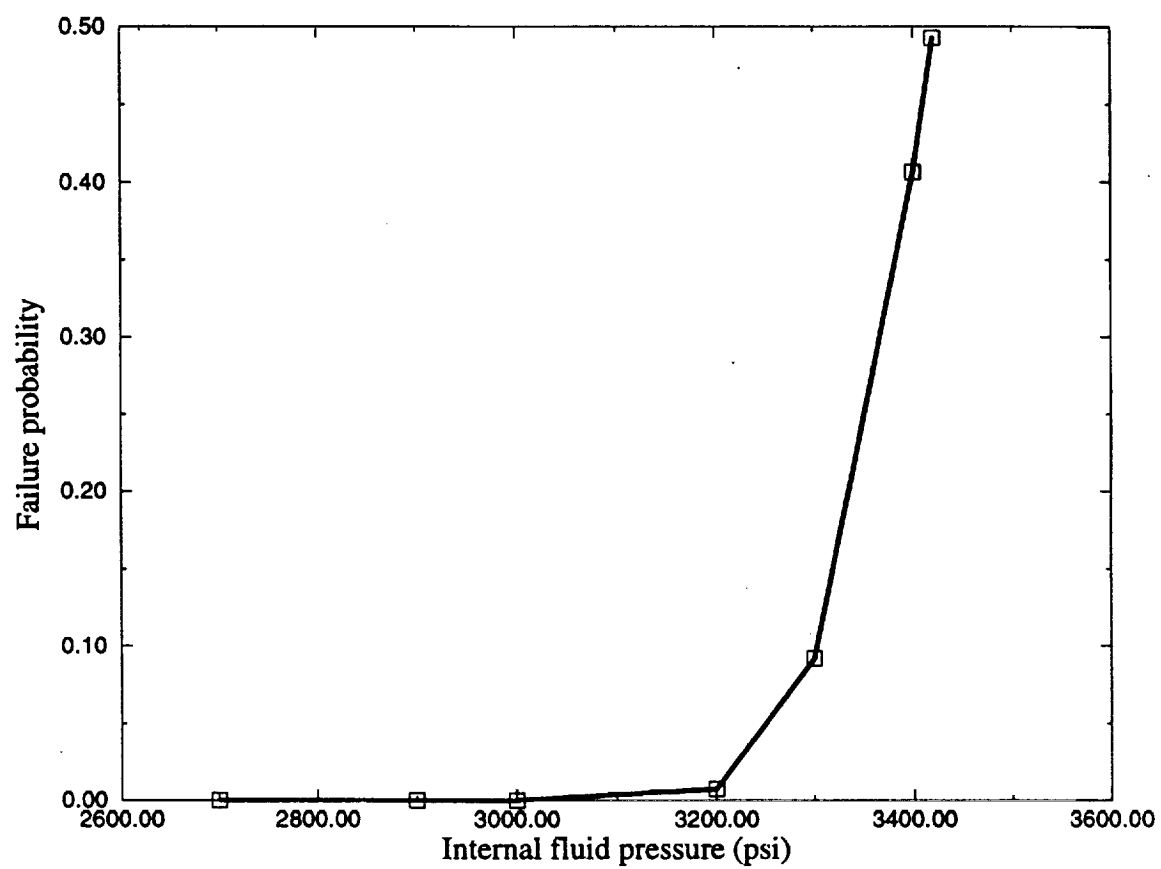


Figure 26: Structural failure probability for various pressures

is the number of limit states ($n = 7$ in this case).

The output files *gfun.mov* provide g-functions defining different limit-states for each level of damage as

$$g_i = \beta_i + \sum_{i=1}^n \alpha_i u_i^* \quad (82)$$

From Table 8, it is clear that temperature variations do not have any significant effect on the structural failure probability. Therefore, only pressure variations are considered as follows:

The parameters provided by *gfun.mov* for the structural limit states corresponding to fluid pressure profile are as follows:

Pressure	β	α_1	α_2
2700.0 psi	6.825696	0.999887	-0.015057
2900.0 psi	5.721588	0.999886	-0.015088
3000.0 psi	4.619647	0.999886	-0.015071
3200.0 psi	2.423429	0.999887	-0.015038
3300.0 psi	1.328801	0.999887	-0.015021
3400.0 psi	0.236641	0.999888	-0.014959
3420.0 psi	0.018494	0.999887	-0.015003

The above data provides parameters for 7 g-functions. Using the above data to calculate the union of the region defined by a group of g-functions [Ang and Tang, 1984], we obtain the probability of structural failure involving the progressive damage caused by fluid pressure. The probability is defined by a lower and upper bounds, which in this case are both 0.4926211. The data and output files are shown in

Appendix G. Those bounds provide the overall structural failure estimation when the system experiences various levels of progressive damage. The fluid pressure changes in a range from 2700.0 *psi* to 3420.0 *psi*.

The above two methods provide practical tools for multi-disciplinary system reliability estimation using NESSUS. With multiple impositions of one mode on the other mode, a close approximation to the failure domain can be constructed, and the critical failure probability can be obtained through the union of the failure region defined by the various limit-states.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

Conclusion

This report has demonstrated the application of equivalence concepts to the reliability analysis of multi-disciplinary systems using NESSUS. A thermal-structural-fluid system is used to illustrate the proposed methodology. The analogous model is a very powerful tool to analyze the one-dimensional steady state problem in heat transfer and fluid mechanics by converting those models into a structural model. Then the NESSUS probability analysis program can be implemented and the precise system reliability can be evaluated. Both traditional and progressive system failure probability methods using NESSUS provide practical tools for multi-disciplinary system reliability analysis.

Recommendations for future research

This research project demonstrated how the NESSUS program could be applied for reliability analysis of engineering systems involving different disciplines, such as structure, heat transfer and fluid mechanics. The current models are based on the condition of one-dimensional, steady-state for both heat transfer and fluid mechanics. More complex systems could be treated in the similar way. However, the scope of application of this methodology is largely dependent on the ability of NESSUS/FEM to deal with problems in different disciplines under more complicated situations, for

instance, a thermal or a flow model in two or three dimensions and non-steady state conditions. For more sophisticated systems, the need for separate FEM program may be inevitable. Either a new source code should be developed or the existing commercial softwares could be integrated into the program. Several users of NESSUS have already integrated its FPI module to other FEM analysis program such as ANSYS and NASTRAN. Nevertheless, the use of equivalent concepts helps to obtain a quick estimate of multi-disciplinary system reliability through the use of NESSUS.

The idea of progressive damage imposition to quantify nonlinear system reliability effects has previously been applied to structural mechanics problems [1]. This study extends this concept to multi-disciplinary systems. This appears to be a practical methodology for system reliability analysis when failure modes (even in different disciplines) have physical relationship with each other. The methodology should be pursued further for application to other, more complicated engineering systems.

Appendix A

FEM FILES FOR HEAT TRANSFER OF A HEAT EXCHANGER

(Refer to Chapter III, Fig.16)

NESSUS/FEM input file

```
exp2fem.dat      Wed May 14 20:37:04 1997      1

*FEM
C .... HEAT TRANSFER FOR A REFRIGERANT PIPE
C
C
C .... PARAMETER DATA
C
*BOUN      20
*CONS      0
*DISP
*NODE      6
*DOPL      4
*ELEM      3
98
*PORC      1
*PERT      4  4
*PRIN
*BEAM
*END
C ***** MODEL DATA
*COORD
1      0.0000  0.0000  0.0000  0.0000  0.0000  1.0000
2      0.0000  10.0000  0.0000  0.0000  0.0000  1.0000
3      0.0000  10.0000  0.0000  0.0000  0.0000  1.0000
4      0.0000  20.0000  0.0000  0.0000  0.0000  1.0000
5      0.0000  20.0000  0.0000  0.0000  0.0000  1.0000
6      0.0000  30.0000  0.0000  0.0000  0.0000  1.0000
*ELEM      98
1      1      2
2      3      4
3      5      6
*DOPL
3      2
5      4
*BOUN
6      1  0.0000
6      3  0.0000
1      1  0.0000
1      2  70.0000
1      3  0.0000
1      4  0.0000
1      5  0.0000
1      6  0.0000
*BEAM
1      6  1.0000  1.0000
*ITERA
40      0.0500
*PROP      98
1      2  1.0000  69.63867  0.3000  0.0000  2.5E-4
3      4  1.0000  79.9230  0.3000  0.0000  2.5E-4
5      6  1.0000  39955.5476  0.3000  0.0000  2.5E-4
*PORC      6
6      2  -241.66
*PRIN
*END      TOTA  NODE
```

NESSUS/FEM output file

exp2fem.out Wed May 14 20:50:14 1997 1

1

```

NN  NN  EEEEEEE  SSSSSSS  SSSSSSS  UU  UU  SSSSSSS
NNN  NN  EE  SS  SS  UU  UU  SS
NN  N  NN  EE  SS  SS  UU  UU  SS
NN  N  NN  EEEEE  SSSSSSS  SSSSSSS  UU  UU  SSSSSS
NN  N  NN  EE  SS  SS  UU  UU  SS
NN  NNN  EE  SS  SS  UU  UU  SS
NN  NN  EEEEEEE  SSSSSSS  SSSSSSS  UUUUU  SSSSSSS

```

```

NNNNN  NNN  FFFFFFFF  EEEEEEE  MM  MM
N  N  N  FF  EE  MM  MM
NN  NN  NN  FF  EE  MM  N  MM
NNNNNN  FFFFFFFF  EEEEEEE  MM  N  MM
NNN  NN  NN  FF  EE  MM  MM
N  N  N  FF  EE  MM  MM
N  N  N  FF  EEEEEEE  MM  MM
N  N  N

```

DATE: 14- 5-1997 20:50 - LEVEL 6.0 (109) - DATED June, 30 , 1992

C ... HEAT TRANSFER FOR A REGRIGERANT PIPE

 ** PERTURBATION NO. 4 **

SUMMARY OF PERTURBATION DATA

VARIABLE NO.	1	MEAN VALUE	0.00000D+00	STANDARD DEVIATION	0.00000D+00
PERTURBED VALUE	0.00000D+00				
VARIABLE NO.	2	MEAN VALUE	0.00000D+00	STANDARD DEVIATION	0.00000D+00
PERTURBED VALUE	0.00000D+00				
VARIABLE NO.	3	MEAN VALUE	0.00000D+00	STANDARD DEVIATION	0.00000D+00
PERTURBED VALUE	0.00000D+00				
VARIABLE NO.	4	MEAN VALUE	0.00000D+00	STANDARD DEVIATION	0.00000D+00
PERTURBED VALUE	0.00000D+00				

PERTURBATION ASSEMBLY, INCR. 0 PERT. 4 ITER. 0 CPTIME= 0.31 SEC

*** PERTURBATION 4 HAS NO EFFECT ON THIS ANALYSIS

1
 DATABASE UPDATE, INCR. 0 PERT. 4 ITER. 0 CPTIME= 0.32 SEC
 1C ... HEAT TRANSFER FOR A REGRIGERANT PIPE DATE
 : 14- 5-1997 20:50 RESULTS PAGE: 1

0 TOTAL DISPLACEMENTS INCREMENT 0 TOTAL TRANSIENT TIME 0.00000D+0

NODE COMP. 6	COMP. 1	COMP. 2	COMP. 3	COMP. 4	COMP. 5
1	0.00000D+00	0.70000D+02	0.00000D+00	0.00000D+00	0.00000D+00
2	0.00000D+00	0.35298D+02	0.00000D+00	0.00000D+00	0.00000D+00
3	0.00000D+00	0.35298D+02	0.00000D+00	0.00000D+00	0.00000D+00
4	0.00000D+00	0.50614D+01	0.00000D+00	0.00000D+00	0.00000D+00
5	0.00000D+00	0.50614D+01	0.00000D+00	0.00000D+00	0.00000D+00
6	0.00000D+00	0.50009D+01	0.00000D+00	0.00000D+00	0.00000D+00

1C ... HEAT TRANSFER FOR A REGRIGERANT PIPE VERSION 6.0(109)
) DATE: 14- 5-1997 20:50 PAGE: 11

END OF INCREMENT, INCR. 0 PERT. 0 ITER. 0 CPTIME= 0.34 SEC
 1C ... HEAT TRANSFER FOR A REGRIGERANT PIPE VERSION 6.0(109)
) DATE: 14- 5-1997 20:50 PAGE: 12

Appendix B

FEM FILES FOR FLUID FLOW IN A DUCT

(Refer to Chapter III, Fig.18)

NESSUS/FEM input file

```

flowfem1.dat          Sat Jun 28 22:26:43 1997          1
*FEM
C .... FLOW IN A CONSTANT DIAMETER PIPE (with A/L=0.1)
C
C
C .... PARAMETER DATA
C
*BOUND      20
*CONST      0
*DISP
*MODE      2
*ELEM      1
*ELEM      98
*FORC      1
*PERT      4 4
*PRIN
*BEAM
*END
C**** MODEL DATA
*COORD
1      0.0000  0.0000  0.0000  0.0000  0.0000  1.0000
2      0.0000  10.000  0.0000  0.0000  0.0000  1.0000
*ELEM      98      1      2
*BOUND
2      1  0.0000
2      3  0.0000
1      1  0.0000
1      2  1838.7
1      3  0.0000
1      4  0.0000
1      5  0.0000
1      6  0.0000
*BEAM
1      2  1.0000  1.0000
*ITERA
0      4
40     0.0500
*PROP      98
1      2  1.0000  14.8960  0.3000  0.0000  2.5E-4
*FORC
2      2  -240.25
*PRIN
TOTA  NODE
*END

```

NESSUS/FEM output file

flowfem1.out Sat Jun 28 22:27:00 1997 4

DATE: 28- 6-1997 22:26 PAGE: 9

** PERTURBATION NO. 4 **

SUMMARY OF PERTURBATION DATA

VARIABLE NO.	1	MEAN VALUE	0.00000D+00	STANDARD DEVIATION	0.00000D+00
PERTURBED VALUE	0.00000D+00				
VARIABLE NO.	2	MEAN VALUE	0.00000D+00	STANDARD DEVIATION	0.00000D+00
PERTURBED VALUE	0.00000D+00				
VARIABLE NO.	3	MEAN VALUE	0.00000D+00	STANDARD DEVIATION	0.00000D+00
PERTURBED VALUE	0.00000D+00				
VARIABLE NO.	4	MEAN VALUE	0.00000D+00	STANDARD DEVIATION	0.00000D+00
PERTURBED VALUE	0.00000D+00				

PERTURBATION ASSEMBLY, INCR. 0 PERT. 4 ITER. 0 CPTIME= 0.27 SEC

*** PERTURBATION 4 HAS NO EFFECT ON THIS ANALYSIS

DATABASE UPDATE, INCR. 0 PERT. 4 ITER. 0 CPTIME= 0.27 SEC
1C ... FLOW IN A CONSTANT DIAMETER PIPE (with A/L=0.1) DAT
E: 28- 6-1997 22:26 RESULTS PAGE: 1

TOTAL DISPLACEMENTS INCREMENT 0 TOTAL TRANSIENT TIME 0.00000D+00

NODE	COMP. 1	COMP. 2	COMP. 3	COMP. 4	COMP. 5
1	0.00000D+00	0.18387D+04	0.00000D+00	0.00000D+00	0.00000D+00
2	0.00000D+00	0.16774D+04	0.00000D+00	0.00000D+00	0.00000D+00

1C ... FLOW IN A CONSTANT DIAMETER PIPE (with A/L=0.1) VERSION 6.0(10)
9) DATE: 28- 6-1997 22:26 PAGE: 10

END OF INCREMENT, INCR. 0 PERT. 0 ITER. 0 CPTIME= 0.29 SEC
1C ... FLOW IN A CONSTANT DIAMETER PIPE (with A/L=0.1) VERSION 6.0(10)
9) DATE: 28- 6-1997 22:26 PAGE: 11

45

STOP DUE TO END OF INPUT FILE

Appendix C

PFEM FILES FOR STRUCTURAL RELIABILITY

(Chapter IV, Data is shown in Fig.16 of Chapter III)

NESSUS/PFEM input and output files for single Z-level

```

exp2spfm.dat      Tue Jun 24 18:13:00 1997      1

*PFEM
C ... PROBABILISTIC ANALYSIS FOR STRUCTURAL RELIABILITY OF EXP2
*NODEFINE
  *COND      0
  *DATATYPE  0
  *RESPTYPE  3
  *COMP      3
  *NODE      44
  *PERT      3
  1,2,3
  *RANVAR    3
  1,2,3
*END
*2PDEFINE
  *COMPUTATIONAL 1 3
  1,2,3
*END
*RVDEFINE
  *DEFINE 1
  FI
  1577.4 315.48 NORMAL
  PRESSURE
  30 58 1.0
  *DEFINE 2
  EMOD
  0.1700000E+08 0.1700000E+07 NORMAL
  PROP 153
  1 58 0.0 1.0 0.0 0.0 0.0
  *DEFINE 3
  ALFA
  0.9500000E-05 0.1900000E-05 NORMAL
  PROP 153
  1 58 0.0 0.0 0.0 1.0 0.0
  *PERT 1
  1 0.1
  *PERT 2
  2 0.1
  *PERT 3
  3 0.1
*END
*END
C *****
*FEM
C ... REFRIGERANT PIPE UNDER INSIDE PRESURE
C
C
C
C ... PARAMETER DATA
C
  *CONS 0
  *DISP
  *NODE 58
  *ELEM 28
  153
  *BOON 20
  *PRESSURE
  *TEMPERATURE
  *LOAD 2 1 3 1
  *PERT 10 10
  PRESSURE
  PROPERTY
  PROPERTY
*PRIN
*END
C**** MODEL DATA
*COORD
  1 -5.000 0.95

```

2	-4.500	0.95
3	-4.000	0.95
4	-3.500	0.95
5	-3.000	0.95
6	-2.700	0.95
7	-2.400	0.95
8	-2.100	0.95
9	-1.800	0.95
10	-1.500	0.95
11	-1.200	0.95
12	-0.900	0.95
13	-0.600	0.95
14	-0.300	0.95
15	0.000	0.95
16	0.300	0.95
17	0.600	0.95
18	0.900	0.95
19	1.200	0.95
20	1.500	0.95
21	1.800	0.95
22	2.100	0.95
23	2.400	0.95
24	2.700	0.95
25	3.000	0.95
26	3.500	0.95
27	4.000	0.95
28	4.500	0.95
29	5.000	0.95
30	-5.000	0.665
31	-4.500	0.665
32	-4.000	0.665
33	-3.500	0.665
34	-3.000	0.665
35	-2.700	0.665
36	-2.400	0.665
37	-2.100	0.665
38	-1.800	0.665
39	-1.500	0.665
40	-1.200	0.665
41	-0.900	0.665
42	-0.600	0.665
43	-0.300	0.665
44	0.000	0.665
45	0.300	0.665
46	0.600	0.665
47	0.900	0.665
48	1.200	0.665
49	1.500	0.665
50	1.800	0.665
51	2.100	0.665
52	2.400	0.665
53	2.700	0.665
54	3.000	0.665
55	3.500	0.665
56	4.000	0.665
57	4.500	0.665
58	5.000	0.665
*ELEM 153		
1	1	30 31 2
2	2	31 32 3
3	3	32 33 4
4	4	33 34 5
5	5	34 35 6
6	6	35 36 7
7	7	36 37 8
8	8	37 38 9
9	9	38 39 10

exp2spfm.dat

Tue Jun 24 18:13:00 1997

2

```

10      10      39      40      11
11      11      40      41      12
12      12      41      42      13
13      13      42      43      14
14      14      43      44      15
15      15      44      45      16
16      16      45      46      17
17      17      46      47      18
18      18      47      48      19
19      19      48      49      20
20      20      49      50      21
21      21      50      51      22
22      22      51      52      23
23      23      52      53      24
24      24      53      54      25
25      25      54      55      26
26      26      55      56      27
27      27      56      57      28
28      28      57      58      29

*PRESSURE
30      58      1577.4
*TEMPERATURE
1      29      4.06
30      58      4.00
*BOUN
15      1      0.0
44      1      0.0
*ITERA
40      1      3
40      0.0500
*PROP
153
1      58      0.1000E-01      0.1700E-08      0.140      0.9500E-05      0.3168      6.0E+06
*PRIN
STAR
NODE
*END
C *****
*FPI
HEAT EXCHANGER OF EXP2 WITH DATASETS
*RVNOM
3
*OFUNCTION
1
*DASETS
4
*METHOD
1
*PRINTOPT
0
*ANALTYPE
1
*END
*ELEVELS
1
0.800000E+04
*END

```

DESIGN SENSITIVITIES

TAYLOR SERIES EXPANSION OF THE FORM

$$G = A_0 + A_1 \cdot X_1 + A_2 \cdot X_2 + \dots + A_N \cdot X_N$$

WHERE:

exp2spfm.out

Tue Jun 24 18:13:42 1997

3

A0 IS THE CONSTANT TERM
A1, A2, ..., AN ARE THE DESIGN SENSITIVITIES
X1, X2, ..., XN ARE THE RANDOM VARIABLES

TAYLOR SERIES COEFFICIENTS A COEFFS.	VALUE	RANDOM VARIABLE	EXPANSION POINT (NPP)	NORM. DESIGN SENSITIVITY
0	0.209646E+04			
1	0.233334E+01	1	0.157740E+04	0.635143E+00
2	-0.121121E-03	2	0.170000E+08	-0.361809E+00
3	0.168750E+07	3	0.950000E-05	0.276730E-02

PERFORMING PROBABILISTIC ANALYSIS WITH FPI

Z	U	PROBABILITY	ITER. NO.
0.80000000E+04	0.36223765E+01	0.99999999E+00	0

NESSUS/PFEM input and output file for CDF

```

exp2scdf.dat      Tue Jun 24 19:19:57 1997      1

*PFEM
C ... PROBABILISTIC ANALYSIS FOR STRUCTURAL RELIABILITY OF EXP2
*NODEDEFINE
  *COND      0
  *DATATYPE  0
  *RESPTYPE  3
  *COMP      3
  *NODE      44
  *PERT      3
  1,2,3
  *RANVAR    3
  1,2,3
*END
*EFDDEFINE
  *COMPUTATIONAL 1 3
  1,2,3
*END
*RVDEFINE
  *DEFINE 1
  FI
  0.13774000E+04 0.3154800E+03 NORMAL
  PRESSURE
  30 58 1.0
  *DEFINE 2
  ENOD
  0.1700000E+08 0.1700000E+07 NORMAL
  PROP 153
  1 58 0.0 1.0 0.0 0.0 0.0
  *DEFINE 3
  ALFA
  0.9300000E-05 0.1900000E-03 NORMAL
  PROP 153
  1 58 0.0 0.0 0.0 1.0 0.0
  *PERT 1
  1 0.1
  *PERT 2
  2 0.1
  *PERT 3
  3 0.1
  *END
C .....
*FEM
C ... REFRIGERANT PIPE UNDER INSIDE PRESURE
C
C
C ... PARAMETER DATA
C
*CONS      0
*DISP
*NODE      58
*ELEM      28
  153
*BOUND     20
*PRESSURE
*TEMPERATURE
*LOAD      2 1 3 1
*PERT      10 10
  PRESSURE
  PROPERTY
  PROPERTY
*PRIN
*END
C**** MODEL DATA
*COOR
  1 -5.000 0.95

```

2	-4.500	0.95
3	-4.000	0.95
4	-3.500	0.95
5	-3.000	0.95
6	-2.700	0.95
7	-2.400	0.95
8	-2.100	0.95
9	-1.800	0.95
10	-1.500	0.95
11	-1.200	0.95
12	-0.900	0.95
13	-0.600	0.95
14	-0.300	0.95
15	0.000	0.95
16	0.300	0.95
17	0.600	0.95
18	0.900	0.95
19	1.200	0.95
20	1.500	0.95
21	1.800	0.95
22	2.100	0.95
23	2.400	0.95
24	2.700	0.95
25	3.000	0.95
26	3.500	0.95
27	4.000	0.95
28	4.500	0.95
29	5.000	0.95
30	-5.000	0.665
31	-4.500	0.665
32	-4.000	0.665
33	-3.500	0.665
34	-3.000	0.665
35	-2.700	0.665
36	-2.400	0.665
37	-2.100	0.665
38	-1.800	0.665
39	-1.500	0.665
40	-1.200	0.665
41	-0.900	0.665
42	-0.600	0.665
43	-0.300	0.665
44	0.000	0.665
45	0.300	0.665
46	0.600	0.665
47	0.900	0.665
48	1.200	0.665
49	1.500	0.665
50	1.800	0.665
51	2.100	0.665
52	2.400	0.665
53	2.700	0.665
54	3.000	0.665
55	3.500	0.665
56	4.000	0.665
57	4.500	0.665
58	5.000	0.665
*ELEM 153		
1	1 30 31 2	
2	2 31 32 3	
3	3 32 33 4	
4	4 33 34 5	
5	5 34 35 6	
6	6 35 36 7	
7	7 36 37 8	
8	8 37 38 9	
9	9 38 39 10	

exp2scdf.dat

Tue Jun 24 19:19:57 1997

2

```

10      10      39      40      11
11      11      40      41      12
12      12      41      42      13
13      13      42      43      14
14      14      43      44      15
15      15      44      45      16
16      16      45      46      17
17      17      46      47      18
18      18      47      48      19
19      19      48      49      20
20      20      49      50      21
21      21      50      51      22
22      22      51      52      23
23      23      52      53      24
24      24      53      54      25
25      25      54      55      26
26      26      55      56      27
27      27      56      57      28
28      28      57      58      29

*PRESSURE
  10      58      0.1577400E+04
*TEMPERATURE
  1      29      4.06
  10      58      4.00
*BOUND
  13      1      0.0
  44      1      0.0
*ITERA      1      3
  40      0.0500
*PROP      153
  1      58      0.1000E+01      0.1700E+08      0.140      0.9500E-05      0.3168      6.0E+06
*PRIN
  STRA      NODE
*END
C .....
*FPI
  HEAT EXCHANGER OF EXP2 WITH DATASETS
*RVNUM 3
*OFUNCTION 1
*DATASETS 4
*METHOD 1
*PRINTOPT 0
*ANALTYPE 0
*END
*END

```

DESIGN SENSITIVITIES

TAYLOR SERIES EXPANSION OF THE FORM

$$G = A_0 + A_1 \cdot X_1 + A_2 \cdot X_2 + \dots + A_N \cdot X_N$$

WHERE:

exp2scdf.out

Tue Jun 24 18:28:49 1997

3

A0 IS THE CONSTANT TERM
A1, A2, ..., AN ARE THE DESIGN SENSITIVITIES
X1, X2, ..., XN ARE THE RANDOM VARIABLES

TAYLOR SERIES COEFFICIENTS		RANDOM		EXPANSION	NORM. DESIGN
A COEFFS.	VALUE	VARIABLE	POINT (MPP)		SENSITIVITY
0	0.209646E+04				
1	0.233334E+01	1	0.157740E+04	0.635343E+00	
2	-0.123321E-03	2	0.170000E+08	-0.361889E+00	
3	0.168750E+07	3	0.950000E-05	0.276730E-02	

PERFORMING PROBABILISTIC ANALYSIS WITH FPI

CDF RESULTS		PROBABILITY		ITER. NO.
X	U			
-0.20689941E+03	-0.51000000E+01	0.17012231E+06		0
0.55849989E+03	-0.41000000E+01	0.20668716E+04		0
0.13218992E+04	-0.31000000E+01	0.96767122E-03		0
0.20692985E+04	-0.21000000E+01	0.17864357E-01		0
0.28546978E+04	-0.11000000E+01	0.13566610E+00		0
0.43854964E+04	0.90000000E+00	0.81593991E+00		0
0.51508957E+04	0.19000000E+01	0.97128351E+00		0
0.59162950E+04	0.29000000E+01	0.99813412E+00		0
0.66816943E+04	0.39000000E+01	0.99995188E+00		0
0.74470935E+04	0.49000000E+01	0.99999952E+00		0

Appendix D

PFEM FILES FOR THE HEAT TRANSFER RELIABILITY

(Chapter IV, data is shown in Fig.16 in Chapter III)

NESSUS/PFEM input and output files for single Z-level

```
exp2tempfemx.dat          Sun Jun 22 01:05:39 1997          1

*PFEM
C ... PROBABILISTIC ANALYSIS FOR TEMPERATURE FAILURE OF EXP2 2-LEVEL
*NVDEFINE
  *COND      0
  *DATATYPE  0
  *RESPTYPE  1
  *COMP      2
  *NODE      6
  *PERT      3
  1,2,3
  *RANVAR    3
  1,2,3
*END
*ZFDEFINE
  *COMPUTATIONAL 1 3
1,2,3
*END
*KVDEFINE
  *DEFINE 1
EMOD1
0.39955476000E+05 0.39955476000E+04 NORMAL
PROP 98
5 6 0.0 1.0 0.0 0.0 0.0
  *DEFINE 2
EMOD2
0.588758000E+02 0.588758000E+01 NORMAL
PROP 98
3 4 0.0 1.0 0.0 0.0 0.0
  *DEFINE 3
EMOD1
0.785398E+02 0.785398E+01 NORMAL
PROP 98
1 2 0.0 1.0 0.0 0.0 0.0
*PERT 1
1 0.1
*PERT 2
2 0.1
*PERT 3
3 0.1
*END
C .....
*PFEM
C ... HEAT TRANSFER FOR A REFRIGERANT PIPE EXP2
C
C
C ... PARAMETER DATA
C
*BOUN 20
*CONS 0
*DISP
*NODE 6
*DUPL 4
*ELEM 3
98
*PORC 1
*PERT 10
PROPERTIES
PROPERTIES
PROPERTIES
*PRIN
*SEAM
*END
C**** MODEL DATA
*COORD
1 0.0000 0.0000 0.0000 0.0000 0.0000 1.0000
```

```

2      0.0000 10.0000 0.0000 0.0000 0.0000 1.0000
3      0.0000 10.0000 0.0000 0.0000 0.0000 1.0000
4      0.0000 20.0000 0.0000 0.0000 0.0000 1.0000
5      0.0000 20.0000 0.0000 0.0000 0.0000 1.0000
6      0.0000 30.0000 0.0000 0.0000 0.0000 1.0000
*KLEM 98
1      1      2
2      3      4
3      5      6
*DOPL
3      2
5      4
*BOUN
6      1      0.0000
6      3      0.0000
1      1      0.0000
1      2      70.0000
1      3      0.0000
1      4      0.0000
1      5      0.0000
1      6      0.0000
*BEAM
1      6      1.0000 1.0000
*ITERA
1      3
40     0.0500
*PROP 98
1      2      1.0000 78.5398 0.1000 0.0000 2.5E-4
3      4      1.0000 58.8758 0.1000 0.0000 2.5E-4
5      6      1.0000 39955.476 0.1000 0.0000 2.5E-4
*FORC
6      2      -241.66
*PRIN
      TOTA  NODE
*END
C *****
*FPI
      REFRIGERANT PIPE RELIABILITY WITH Z-LEVEL(T < 5°F)
*RVNUM 3
*GFUNCTION 1
*DATASETS 4
*METHOD 1
*PRINTOPT 0
*ANALTYPE 1
*END
*ZLEVELS 1
5.00000
*END

```

exp2tempfemz.out

Sun Jun 22 01:06:16 1997

3

A0 IS THE CONSTANT TERM
 A1, A2, ... , AN ARE THE DESIGN SENSITIVITIES
 X1, X2, ... , XN ARE THE RANDOM VARIABLES

TAYLOR SERIES COEFFICIENTS A COEFFS.	VALUE	RANDOM VARIABLE	EXPANSION POINT (MPP)	NORM. DESIGN SENSITIVITY
0	-0.737506E+03			
1	0.151174E+05	1	0.399555E+05	0.841489E-03
2	0.697158E+00	2	0.588758E+02	0.571068E+00
3	0.391765E+00	3	0.785398E+02	0.428090E+00

PERFORMING PROBABILISTIC ANALYSIS WITH FPI

CDF RESULTS			
Z	U	PROBABILITY	ITER. NO.
0.30000000E+01	0.13402715E+01	0.90992140E+00	0

NESSUS/PFEM input and output files for CDF

```

exp1tempfem.dat          Thu Apr  6 17:14:48 1993          1

*PFEM
C ... PROBABILISTIC (CDF) ANALYSIS FOR TEMPERATURE FAILURE OF EXP1
*RVDEFINE
  *COORD      0
  *DATATYPE   0
  *RESPTYPE   1
  *COMP       2
  *MODE       6
  *PERT       3
  1,2,3
  *RANVAR     3
  1,2,3
*END
*ZFDEFINE
  *COMPUTATIONAL 1 1
  1,2,3
*END
*RVDEFINE
  *DEFINE 1
EMOD1
0.39955476000E+05 0.39955476000E+05 NORMAL
PROP 38
5 6 0.0 1.0 0.0 0.0 0.0
  *DEFINE 2
EMOD2
0.588758000E+02 0.588758000E+01 NORMAL
PROP 38
3 4 0.0 1.0 0.0 0.0 0.0
  *DEFINE 3
EMOD1
0.785398E+02 0.785398E+01 NORMAL
PROP 38
1 2 0.0 1.0 0.0 0.0 0.0
  *PERT 1
1 0.1
  *PERT 2
2 0.1
  *PERT 3
3 0.1
*END
*END
C .....
*PFEM
C ... HEAT TRANSFER FOR A REFRIGERANT PIPE EXP1
C
C
C ... PARAMETER DATA
C
  *BOUN 20
  *CONS 0
  *DISP 6
  *NODE 6
  *DUPL 4
  *ELEM 3
  38
  *FORC 1
  *PERT 10 10
  PROPERTIES
  PROPERTIES
  PROPERTIES
  *PRIN
  *BEAM
  *END
C**** MODEL DATA
*COORD
  1 0.0000 0.0000 0.0000 0.0000 0.0000 1.0000

```

```

      2      0.0000 10.0000 0.0000 0.0000 0.0000 1.0000
      3      0.0000 10.0000 0.0000 0.0000 0.0000 1.0000
      4      0.0000 20.0000 0.0000 0.0000 0.0000 1.0000
      5      0.0000 20.0000 0.0000 0.0000 0.0000 1.0000
      6      0.0000 30.0000 0.0000 0.0000 0.0000 1.0000
*ELEM 98
      1      1      2
      2      3      4
      3      5      6
*DUPL
      3      2
      5      4
*BOUN
      6      1      0.0000
      6      3      0.0000
      1      1      0.0000
      1      2      70.0000
      1      3      0.0000
      1      4      0.0000
      1      5      0.0000
      1      6      0.0000
*BEAM
      1      6      1.0000 1.0000
*ITERA
      1      3
      40      0.0500
*PROP 98
      1      2      1.0000 78.5398 0.1000 0.0000 2.5E-4
      3      4      1.0000 58.8758 0.1000 0.0000 2.5E-4
      5      6      1.0000 39955.476 0.1000 0.0000 2.5E-4
*FORC
      6      2      -241.66
*PRIN
      TOTA  NODE
*END
C *****
*FPI
      HEAT EXCHAGER OF EXP1 WITH DATASETS
*RVNUM 3
*GFUNCTION 1
*DASETS 4
*METHOD 1
*PRINTOPT 0
*ANALTYPE 0
*END
*END

```

exp2tempem.out Mon May 19 22:40:36 1997 3

A0 IS THE CONSTANT TERM
A1, A2, ... , AN ARE THE DESIGN SENSITIVITIES
X1, X2, ... , XN ARE THE RANDOM VARIABLES

TAYLOR SERIES COEFFICIENTS		RANDOM	EXPANSION	NORM. DESIGN
A COEFFS.	VALUE	VARIABLE	POINT (MPP)	SENSITIVITY
0	-0.737506E+02			
1	0.151374E-05	1	0.399555E+05	0.841489E-03
2	0.697158E+00	2	0.588758E+02	0.571068E+00
3	0.391765E+00	3	0.785398E+02	0.428090E+00

PERFORMING PROBABILISTIC ANALYSIS WITH FPI

Z	U	PROBABILITY	ITER. NO.
-0.28039092E+02	-0.51000000E+01	0.17012231E-06	0
-0.22908941E+02	-0.41000000E+01	0.20668716E-04	0
-0.17778790E+02	-0.31000000E+01	0.96767122E-03	0
-0.12648639E+02	-0.21000000E+01	0.17864357E-01	0
-0.75184880E+01	-0.11000000E+01	0.13566610E+00	0
0.27418138E+01	0.90000000E+00	0.81593991E+00	0
0.78719648E+01	0.19000000E+01	0.97128351E+00	0
0.13002116E+02	0.29000000E+01	0.99813412E+00	0
0.18132267E+02	0.39000000E+01	0.99995188E+00	0
0.23262418E+02	0.49000000E+01	0.99999952E+00	0

Appendix E

PFEM FILES FOR RELIABILITY OF FLUID FLOW

(Chapter IV, data is show in Fig.18 of Chapter III)

NESSUS/PFEM input and output files for a single Z-level

```

flowpfemx.dat          Sat Jun 28 22:36:52 1997          1

*PFEM
C .... PROBABILISTIC ANALYSIS FOR FLOW FAILURE
*RVDEFINE
  *COND      0
  *DATATYPE  0
  *RESPTYPE  1
  *COMP      2
  *NODE      2
  *PERT      2
  1.2
  *RANVAR    2
  1.2
*END
*ZFDEFINE
  *COMPUTATIONAL  1  2
1.2
*END
*RVDEFINE
  *DEFINE  1
EMOD
14.896  1.4896  NORMAL
PROP  98
  1  2  0.0  1.0  0.0  0.0  0.0
  *DEFINE  2
VELO
-350.0  35.0  NORMAL
FORC
  2  2  1.00
*PERT  1
1  0.1
*PERT  2
2  0.1
*END
*END
C .....
C
*FEM
C ... FLOW IN A CONSTANT DIAMETER PIPE (with A/L=0.1)
C
C
C
C ... PARAMETER DATA
C
*BOUN  20
*CONS  0
*DISP  2
*NODE  2
*ELEM  1
  98
*FORC  1
*PERT  10  10
VELOCITY
FORCE
*PRIN
*BEAM
*END
C*** MODEL DATA
*COOR
  1  0.0000  0.0000  0.0000  0.0000  0.0000  1.0000
  2  0.0000  10.000  0.0000  0.0000  0.0000  1.0000
*ELEM  98
  1  1  2
*BOUN
  2  1  0.0000
  2  3  0.0000
  1  1  0.0000
  1  2  1838.7
  1  3  0.0000

```

```

      1  4  0.0000
      1  5  0.0000
      1  6  0.0000
*BEAM      1
      1  2  1.0000  1.0000
*ITERA      1  2
      40  0.0500
*PROP      98
      1  2  1.0000  14.8960  0.3000  0.0000  2.5E-4
*FORC      2  1  -350.0
*PRIN
      TOTA  NODE
*END
C.....
*FPI
      FLOW FAILURE PROBABILITY WITH Z-LEVEL (P<1677.4psi)
*RVNUM 2
*GFUNCTION 1
*DATASETS 3
*METHOD 1
*PRINTOPT 0
*ANALYTTYPE 1
*END
*ZLEVELS 1
1677.4
*END

```

DESIGN SENSITIVITIES

TAYLOR SERIES EXPANSION OF THE FORM

$$G = A_0 + A_1 \cdot X_1 + A_2 \cdot X_2 + \dots + A_N \cdot X_N$$

WHERE:

A₀ IS THE CONSTANT TERM
A₁, A₂, ... , A_N ARE THE DESIGN SENSITIVITIES
X₁, X₂, ... , X_N ARE THE RANDOM VARIABLES

TAYLOR SERIES COEFFICIENTS		RANDOM VARIABLE	EXPANSION POINT (MFP)	NORM. DESIGN SENSITIVITY
A COEFFS.	VALUE			
0	0.160374E+04			

flowpfemx.out Sat Jun 28 22:37:13 1997 3

1	0.157735E+02	1	0.148960E+02	0.500000E+00
2	0.671321E+00	2	-0.350000E+03	-0.500000E+00

PERFORMING PROBABILISTIC ANALYSIS WITH FPI

CDF RESULTS		PROBABILITY	ITER. NO.
Z	U		
0.16774000E+04	0.22174074E-01	0.98670241E+00	0

NESSUS/PFEM input and output files for CDF

```

flowcdf.dat          Sat Jun 28 22:45:21 1997          1

*PFEM
C .... PROBABILISTIC ANALYSIS FOR FLOW FAILURE
*WVDEFINE
  *COND      0
  *DATATYPE  0
  *RESPTYPE  1
  *COMP      2
  *MODE      2
  *PERT      2
  1.2
  *RAVVAR    2
  1.2
  *END
*ZFDEFINE
  *COMPUTATIONAL 1 2
1.2
*END
*WVDEFINE
  *DEFINE 1
END
14.896 1.4896 NORMAL
PROP 98
1 2 0.0 1.0 0.0 0.0 0.0
  *DEFINE 2
VELO
-350.0 35.0 NORMAL
FORC
  2 2 1.00
*PERT 1
1 0.1
*PERT 2
2 0.1
*END
*END
C .....
*PFEM
C .... FLOW IN A CONSTANT DIAMETER PIPE (with A/L=0.1)
C
C
C
C .... PARAMETER DATA
C
*BOUN 20
*CONS 0
*DISP
*NODE 2
*ELEM 1
98
*FORC 1
*PERT 10 10
VELOCITY
FORCE
*PRIN
*BEAM
*END
C*** MODEL DATA
*COOR
1 0.0000 0.0000 0.0000 0.0000 0.0000 1.0000
2 0.0000 10.000 0.0000 0.0000 0.0000 1.0000
*ELEM 98
1 1 2
*BOUN
2 1 0.0000
2 3 0.0000
1 1 0.0000
1 2 1838.7
1 3 0.0000

```

```

      1  4  0.0000
      1  5  0.0000
      1  6  0.0000
*BEAM  1
      1  2  1.0000  1.0000
*ITERA 1  2
      40  0.0500
*PROP  98
      1  2  1.0000  14.896  0.3000  0.0000  2.5E-4
*FORC  2  2  -350.0
*PRIN  TOTA  NODE
*END
C*****
*FPI
      FLOW FAILURE PROBABILITY WITH A SERIES Z-LEVELS
*RVNUM  2
*GFUNCTION  1
*DATASETS  3
*METHOD  1
*PRINTOPT  0
*ANALYTTYPE  0
*END
*END

```

----- DESIGN SENSITIVITIES -----

TAYLOR SERIES EXPANSION OF THE FORM

$$G = A_0 + A_1 \cdot X_1 + A_2 \cdot X_2 + \dots + A_N \cdot X_N$$

WHERE:

A₀ IS THE CONSTANT TERM
A₁, A₂, ..., A_N ARE THE DESIGN SENSITIVITIES
X₁, X₂, ..., X_N ARE THE RANDOM VARIABLES

TAYLOR SERIES COEFFICIENTS		RANDOM VARIABLE	EXPANSION POINT (MPP)	NORM. DESIGN SENSITIVITY
A COEFFS.	VALUE			
0	0.160374E+04			

flowcdf.out Sat Jun 28 22:46:36 1997 3

1	0.157735E+02	1	0.148960E+02	0.500000E+00
2	0.671321E+00	2	-0.350000E+03	-0.500000E+00

PERFORMING PROBABILISTIC ANALYSIS WITH FPI

CDF RESULTS		PROBABILITY	ITER. NO.
Z	Q		
0.14343166E-04	-0.51000000E+01	0.17012231E-06	0
0.14675365E-04	-0.41000000E+01	0.20668716E-04	0
0.15007564E-04	-0.31000000E+01	0.96767122E-03	0
0.15339763E-04	-0.21000000E+01	0.17864357E-01	0
0.15671961E-04	-0.11000000E+01	0.13566610E+00	0
0.16336159E-04	0.90000000E+00	0.81593991E+00	0
0.16668558E-04	0.19000000E+01	0.97128351E+00	0
0.17000756E-04	0.29000000E+01	0.99813412E+00	0
0.17332955E-04	0.39000000E+01	0.99995188E+00	0
0.17665154E-04	0.49000000E+01	0.99999952E+00	0

Appendix F

GFUN.DAT AND GFUN.MOV FILES OF IMPOSITIONS

(Chapter IV, data is show in Fig.16 of Chapter III)

NESSUS/PFEM input gfun.dat and output gfun.mov files

```

gfun.dat      Tue Jul 1 15:39:34 1997      1

*PFEM
C .... PROBABILISTIC ANALYSIS FOR STRUCTURAL RELIABILITY
*HVDDEFINE
  *COND      0
  *DATATYPE  0
  *RESPTYPE  3
  *COMP      3
  *NODE      44
  *PERT      2
  1,2
  *RANVAR    2
  1,2
*END
*ZFDEFINE
  *COMPUTATIONAL 1 2
1,2
*END
*RVDEFINE
  *DEFINE 1
EMOD
0.17000000E+08 0.17000000E+07 NORMAL
PROP 153
1 58 0.0 1.0 0.0 0.0 0.0
  *DEFINE 2
ALFA
0.95000000E-05 0.19000000E-05 NORMAL
PROP 153
1 58 0.0 0.0 0.0 1.0 0.0
  *PERT 1
1 0.1
  *PERT 2
2 0.1
*END
*END
C .....
*FEM
C .... REFRIGERANT PIPE UNDER INSIDE PRESURE
C
C
C .... PARAMETER DATA
C
*CONS 0
*DISP
*NODE 58
*ELEM 28
153
*BOUND 20
*PRESSURE
*TEMPERATURE
*LOAD 2 1 3 1
*PERT 10 10
PROPERTY
PROPERTY
*PRIN
*END
C .... MODEL DATA
*COORD
1 -5.000 0.95
2 -4.500 0.95
3 -4.000 0.95
4 -3.500 0.95
5 -3.000 0.95
6 -2.700 0.95
7 -2.400 0.95
8 -2.100 0.95
9 -1.800 0.95
10 -1.500 0.95
11 -1.200 0.95
12 -0.900 0.95
13 -0.600 0.95
14 -0.300 0.95
15 0.000 0.95
16 0.300 0.95
17 0.600 0.95
18 0.900 0.95
19 1.200 0.95
20 1.500 0.95
21 1.800 0.95
22 2.100 0.95
23 2.400 0.95
24 2.700 0.95
25 3.000 0.95
26 3.500 0.95
27 4.000 0.95
28 4.500 0.95
29 5.000 0.95
30 -5.000 0.665
31 -4.500 0.665
32 -4.000 0.665
33 -3.500 0.665
34 -3.000 0.665
35 -2.700 0.665
36 -2.400 0.665
37 -2.100 0.665
38 -1.800 0.665
39 -1.500 0.665
40 -1.200 0.665
41 -0.900 0.665
42 -0.600 0.665
43 -0.300 0.665
44 0.000 0.665
45 0.300 0.665
46 0.600 0.665
47 0.900 0.665
48 1.200 0.665
49 1.500 0.665
50 1.800 0.665
51 2.100 0.665
52 2.400 0.665
53 2.700 0.665
54 3.000 0.665
55 3.500 0.665
56 4.000 0.665
57 4.500 0.665
58 5.000 0.665
*ELEM 153
1 1 30 31 2
2 2 31 32 3
3 3 32 33 4
4 4 33 34 5
5 5 34 35 6
6 6 35 36 7
7 7 36 37 8
8 8 37 38 9
9 9 38 39 10
10 10 39 40 11
11 11 40 41 12
12 12 41 42 13
13 13 42 43 14
14 14 43 44 15
15 15 44 45 16
16 16 45 46 17
17 17 46 47 18

```

```

gfun.dat      Tue Jul 1 15:39:34 1997      2

18      18 47 48 19
19      19 48 49 20
20      20 49 50 21
21      21 50 51 22
22      22 51 52 23
23      23 52 53 24
24      24 53 54 25
25      25 54 55 26
26      26 55 56 27
27      27 56 57 28
28      28 57 58 29

*PRESSURE
30      58      3300.0
*TEMPERATURE
1      29      4.06
30      58      4.0
*BOUN
15      1      0.0
44      1      0.0
*ITERA 1      2
40      0.0500
*PROP 153
1      58      0.1000E+01  0.1700E+08  0.340  0.9500E-05  0.3168  6.0E+06
*PRIN STRN NODE
*END
C *****
*FPI
HEAT EXCHAGER OF EXP2 WITH DATASETS
*RVNUM 2
*GFUNCTION 1
*DASETS 3
*METHOD 1
*PRINTOPT 0
*ANALTYPE 1
*END
*ZLEVELS 1
0.8000000E+04
*END

```

```

gfun.mov      Tue Jul 1 15:28:37 1997      1

CONDITION #   NODE   COMPONENT   LAYER
-----
INCR      0      44      3      1

Z MEDIAN, MEAN, STD. DEV.
0.654937E+04  0.654937E+04  0.212524E+03
NUMBER OF PROBABILITY LEVELS 1
LVL, ZO, U, PROB. = 1  0.800000E+04  6.82570  1.000000000
DESIGN POINT COORD., SENSITIVITY FACTORS, STD. DEV. FROM MEAN VALUES
ZO, PROB. = 0.800000E+04  1.000000000
ENOD 0.5397631E+07  0.999887  -6.824923
ALFA 0.9695273E-05  -0.015057  0.102775

*****

CDF RESULTS
Z      U      PROBABILITY      ITER. NO.
0.80000000E+04  0.68256956E+01  0.10000000E+01  0

```

gfun.mov Sun Jun 29 03:16:45 1997 1

CONDITION	#	NODE	COMPONENT	LAYER
INCR	0	44	3	1

Z MEDIAN, MEAN, STD. DEV.
 0.678271E+04 0.678271E+04 0.212754E+03
 NUMBER OF PROBABILITY LEVELS 1
 LVL, Z0, u, PROB. = 1 0.800000E+04 5.72159 0.999999999
 DESIGN POINT COORD., SENSITIVITY FACTORS, STD. DEV. FROM MEAN VALUES
 Z0, PROB. = 0.800000E+04 0.999999999
 EMOD 0.7274407E+07 0.999886 -5.720937
 ALFA 0.9664020E-05 -0.015088 0.086326

.....

Z	U	PROBABILITY	ITER. NO.
0.80000000E+04	0.57215881E+01	0.99999999E+00	0

gfun.mov Sun Jun 29 03:11:36 1997 1

CONDITION	#	NODE	COMPONENT	LAYER
INCR	0	44	3	1

Z MEDIAN, MEAN, STD. DEV.
 0.701604E+04 0.701604E+04 0.212994E+03
 NUMBER OF PROBABILITY LEVELS 1
 LVL, Z0, u, PROB. = 1 0.800000E+04 4.61965 0.999998076
 DESIGN POINT COORD., SENSITIVITY FACTORS, STD. DEV. FROM MEAN VALUES
 Z0, PROB. = 0.800000E+04 0.999998076
 EMOD 0.9147492E+07 0.999886 -4.619122
 ALFA 0.9632282E-05 -0.015071 0.069622

.....

Z	U	PROBABILITY	ITER. NO.
0.80000000E+04	0.46196471E+01	0.99999808E+00	0

gfun.mov Sun Jun 29 03:13:50 1997 1

CONDITION	#	NODE	COMPONENT	LAYER
INCR	0	44	3	1

Z MEDIAN, MEAN, STD. DEV.
 0.748271E+04 0.748271E+04 0.213454E+03
 NUMBER OF PROBABILITY LEVELS 1
 LVL, Z0, u, PROB. = 1 0.800000E+04 2.42143 0.992312628
 DESIGN POINT COORD., SENSITIVITY FACTORS, STD. DEV. FROM MEAN VALUES
 Z0, PROB. = 0.800000E+04 0.992312628
 EMOD 0.1288064E+08 0.999887 -2.421155
 ALFA 0.9569244E-05 -0.015038 0.036444

.....

Z	U	PROBABILITY	ITER. NO.
0.80000000E+04	0.24234292E+01	0.99231263E+00	0

gfun.mov Tue Jul 1 15:39:56 1997 1

CONDITION	#	MODE	COMPONENT	LAYER
INCR	0	44	3	1

Z MEDIAN, MEAN, STD. DEV.
 0.771604E+04 0.771604E+04 0.213694E+03
 NUMBER OF PROBABILITY LEVELS 1
 LVL, ZO, U, PROB. = 1 0.800000E+04 1.32880 0.908043163
 DESIGN POINT COORD., SENSITIVITY FACTORS, STD. DEV. FROM MEAN VALUES
 ZO, PROB. = 0.800000E+04 0.908043163
 EMOD 0.1474129E+08 0.999887 -1.328651
 ALFA 0.9537925E-05 -0.015021 0.019961

.....

Z	U	PROBABILITY	ITER. NO.
0.80000000E+04	0.13288012E+01	0.90804316E+00	0

gfun.mov Tue Jul 1 15:36:47 1997 1

CONDITION	#	MODE	COMPONENT	LAYER
INCR	0	44	3	1

Z MEDIAN, MEAN, STD. DEV.
 0.794938E+04 0.794938E+04 0.213924E+03
 NUMBER OF PROBABILITY LEVELS 1
 LVL, ZO, U, PROB. = 1 0.800000E+04 0.23664 0.593531990
 DESIGN POINT COORD., SENSITIVITY FACTORS, STD. DEV. FROM MEAN VALUES
 ZO, PROB. = 0.800000E+04 0.593531990
 EMOD 0.1659776E+08 0.999888 -0.236614
 ALFA 0.9506724E-05 -0.014959 0.003540

.....

Z	U	PROBABILITY	ITER. NO.
0.80000000E+04	0.23664019E+00	0.59353199E+00	0

gfun.mov Tue Jul 1 16:17:30 1997 1

CONDITION	#	MODE	COMPONENT	LAYER
INCR	0	44	3	1

Z MEDIAN, MEAN, STD. DEV.
 0.799604E+04 0.799604E+04 0.213964E+03
 NUMBER OF PROBABILITY LEVELS 1
 LVL, ZO, U, PROB. = 1 0.800000E+04 0.01849 0.507377567
 DESIGN POINT COORD., SENSITIVITY FACTORS, STD. DEV. FROM MEAN VALUES
 ZO, PROB. = 0.800000E+04 0.507377567
 EMOD 0.1696856E+08 0.999887 -0.018492
 ALFA 0.9500527E-05 -0.015003 0.000277

.....

Z	U	PROBABILITY	ITER. NO.
0.80000000E+04	0.18493758E-01	0.50737757E+00	0

Appendix G

DATA AND OUTPUT FILES FOR CALCULATION OF UNION PROBABILITY

(Chapter IV)

```
temp.dat      Tue Jul  1 16:23:17 1997      1
7, 2, 1
1, 17000000.0,1700000.0
2, 0.0000095,0.0000019
1, 6.825696
2, 5.721588
3, 4.619647
4, 2.423429
5, 1.328801
6, 0.236641
7, 0.018497
1, 6.825696 0.999887 -0.015057
2, 5.721588 0.999886 -0.015088
3, 4.619647 0.999886 -0.015071
4, 2.423429 0.999887 -0.015038
5, 1.328801 0.999887 -0.015021
6, 0.236641 0.999888 -0.014959
7, 0.018497 0.999887 -0.015003
```

```
***** BOUNDS *****
Lower bound = 0.4926211
Upper bound = 0.4926211
```

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